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Reprints from the Early Days of Information Sciences

Paul Ehrenfest – Remarks on Algebra of Logic and Switching Theory

Reprints from History of Information Sciences
Detalji iz istorije informacionih nauka
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情報科学における歴史的論文の復刻
ՎԵՐԱՀՐԱՏԱՐԱԿՈՒՄ ՊԱՏՄՈՒԹՅՈՒՆԻՑ

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Tampere International Center for Signal Processing
TICSP series # 54

Radomir S. Stankovic & Jaakko T. Astola (Eds.)

Reprints from the Early Days of Information Sciences

Paul Ehrenfest – Remarks on Algebra of Logic and Switching Theory

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Paul Ehrenfest - Remarks

on

the Algebra of Logic and Switching Theory

2010

Editors' Notice

This publication has been written and edited by Radomir S. Stanković and Jaakko T. Astola.

Contents

Reprints from The Early Days of Information Sciences

Historical studies about a scientific discipline are a sign of its maturity. When properly understood and carried out, this kind of study is more than an enumeration of facts or giving credit to particular important researchers. It is more a way of discovering and tracing the ways of thinking that have led to important discoveries. In this respect, it is interesting and also important to recall publications where some important concepts, theories, methods, and algorithms were introduced for the first time.

In every branch of science there are some important results published in national or local journals or other publications that have not been widely distributed for different reasons, due to which they often remain unknown to the research community and therefore are rarely referenced. Sometimes the importance of such discoveries is overlooked or underestimated even by the inventors themselves. Such inventions are often re-discovered much later, but their initial sources may remain almost forgotten, and mostly remain sporadically recalled and mentioned within quite limited circles of experts. This is especially often the case with publications in languages other than the English language which is presently the most commonly used language in the scientific world.

This series of publications is aimed at reprinting and, when appropriate, also translating some less known or almost forgotten, but important publications, where some concepts, methods or algorithms were discussed for the first time or introduced independently of other related works.

Another aim of the Reprints is to collect and present in the same place the publications on certain particular subjects of important scholars whose scientific work is signified by contributions to different areas of science.

R.S. Stanković, J.T. Astola

Paul Ehrenfest - Remarks

on

the Algebra of Logic and Switching Theory

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The Editors are grateful to Mrs. Marju Taavetti, the Librarian of the Library of the Tampere University of Technology, and Mrs. Pirkko Ruotsalainen, the Development Manager of the Department of Signal Processing, Tampere University of Technology, for their help in collecting the relevant literature.

Special thanks are due to Ms. Galina Alexandrovna Aukhadieva, Kazan State University, N.I. Lobachevsky Scientific Library, 18, Kremliovskaya Str., Kazan, Russia p/b 420008, for her help in providing the review by P. Ehrenfest that is reprinted in this issue.

The Remarks of Paul Ehrenfest on Algebra of Logic

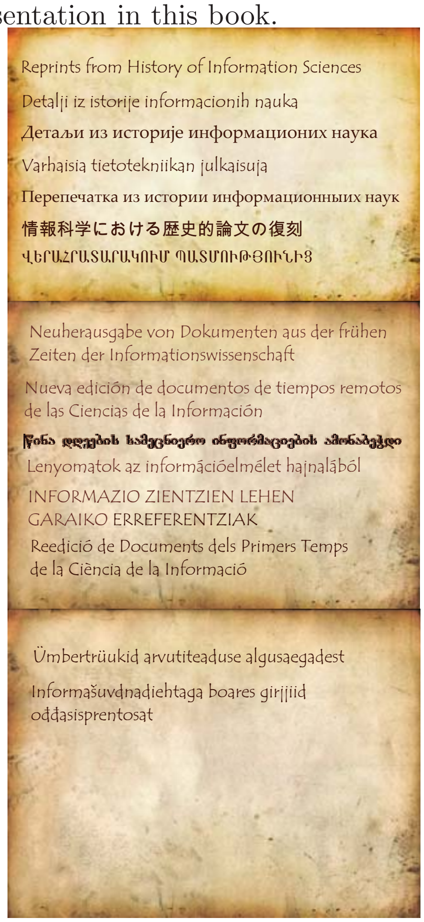
Abstract

The present issue of the Reprints from the Early Days of Information Sciences discusses the remarks of Paul Ehrenfest on the applications of the Algebra of Logic in the design of logic networks. The remarks were made in a review of the Russian edition of the book The Algebra of Logic by Louis Couturat that was published in 1910. This issue contains reprints of the review by Ehrenfest and presents a translation of the review from Russian into English. We believe that this is the first translation of the complete text of this review into English and its first reprint. The remarks by Ehrenfest were mentioned in the reviews of several other publications in the field of logic design. These reviews are also reprinted.

Notice

This book contains several reprints of pages from articles or reviews of articles and books where the remarks of Paul Ehrenfest about applicability of algebra of logic in the design of logic networks were mentioned. These articles and reviews were written by eminent scholars in this field and confirm their knowledge of this work. We did not want to rephrase or rewrite their original statements, since we believe that the way they were presented originally has a particular value for the reader.

We kindly ask for these reprints not to be considered simply as graphic illustrations from previous publications, but to be read as part of the presentation in this book.



English, Serbian Latin, Serbian Cyrillic, Finnish, Russian, Japanese, Armenian, German, Castilian, Georgian, Hungarian, Bask, Estonian, Sami

1 Remarks on the Origins of Switching Theory

Logic networks and many sophisticated techniques for designing them antedate digital computers by many years. The initial applications were in the design of telephone central office equipment. The key concept, which transformed the design process from an art or skills based on the experience of the designers into a science, was the idea of describing both the functions performed and the circuits themselves in terms of Boolean algebra. This observation and related subsequent derivations led to Switching theory as the mathematic foundations for the design of logic networks.

As is is usually the case in engineering and science, a new area or a subdiscipline starts developing by solving first some particular task, with the solution derived based on previous experiences and skills of individuals. If the task is important and the solution efficient and useful, whatever criteria of efficiency and usefulness are, demands for repeated solutions of the same or similar tasks soon arise. Then attempts towards the automatization of the related method or the procedure are naturally made. This necessary requires a formal description of both the problem and the method used to solve it, which requires introduction of certain notions and definitions and leads to the establishing of basic theoretical foundations. Improving performances of solutions and increasing complexity of systems where the task is enrolled, are next to be considered. When the complexity of the system and, therefore, the task, reaches certain level after which it becomes unsolvable by hand and it is hard to produce a solution based just on the experience and skills from practice, some underlying theory is required. Depending on the importance of the problem, formulating such a theory is considered by many scholars in different parts of the world. They are working at about the same time or even simultaneously, however, independently and without knowledge of the work of others. Clearly, researchers might become aware of the related work of others after publication of some results and achievements. After that, authors start referring to the related works of others, as well as try to put their results in a wider context and establish links to the existing related theories.

The development of Switching theory is a typical example of such a scenario of scientific development. In late nineteen thirties, contact and relay networks were widely used in various telecommunication and control systems. The design of these networks was a challenging task requiring a lot of engineering experience and skills. Many researchers had searched for an underlying theory that will enable automatization of the design of such networks, their simplification, and optimization with respect to various criteria. These research efforts lead to establishing Switching theory as mathematical foundations for Logic design involving Boolean algebra as its central part.

Claude Elwood Shannon presented the idea of using Boolean algebra as a kernel of Switching Theory in his Master Thesis defended in 1938 at the Massachusetts Institute of Technology (Fig. 1). In the Thesis, Shannon provided the following references (in the original formulation as in the thesis)

1. "A complete bibliography of the literature of symbolic logic", in *Journal of Symbolic Logic*, Vol. 1, No. 4, December 1936.

2. Louis Couturat, *The Algebra of Logic*, The Open Court Publishing Co.

3. A.N. Whitehead, *Universal Algebra*, Cambridge at the University Press, Vol. I, Book II, Chapters I and II, 35-82.

4. E.V. Huntington, *Transactions of the American Mathematical Society*, Vol. 35, 1933, 274-304.

5. George Boole, *Finite Differences*, G.E. Strechert & Co., Chap. X.

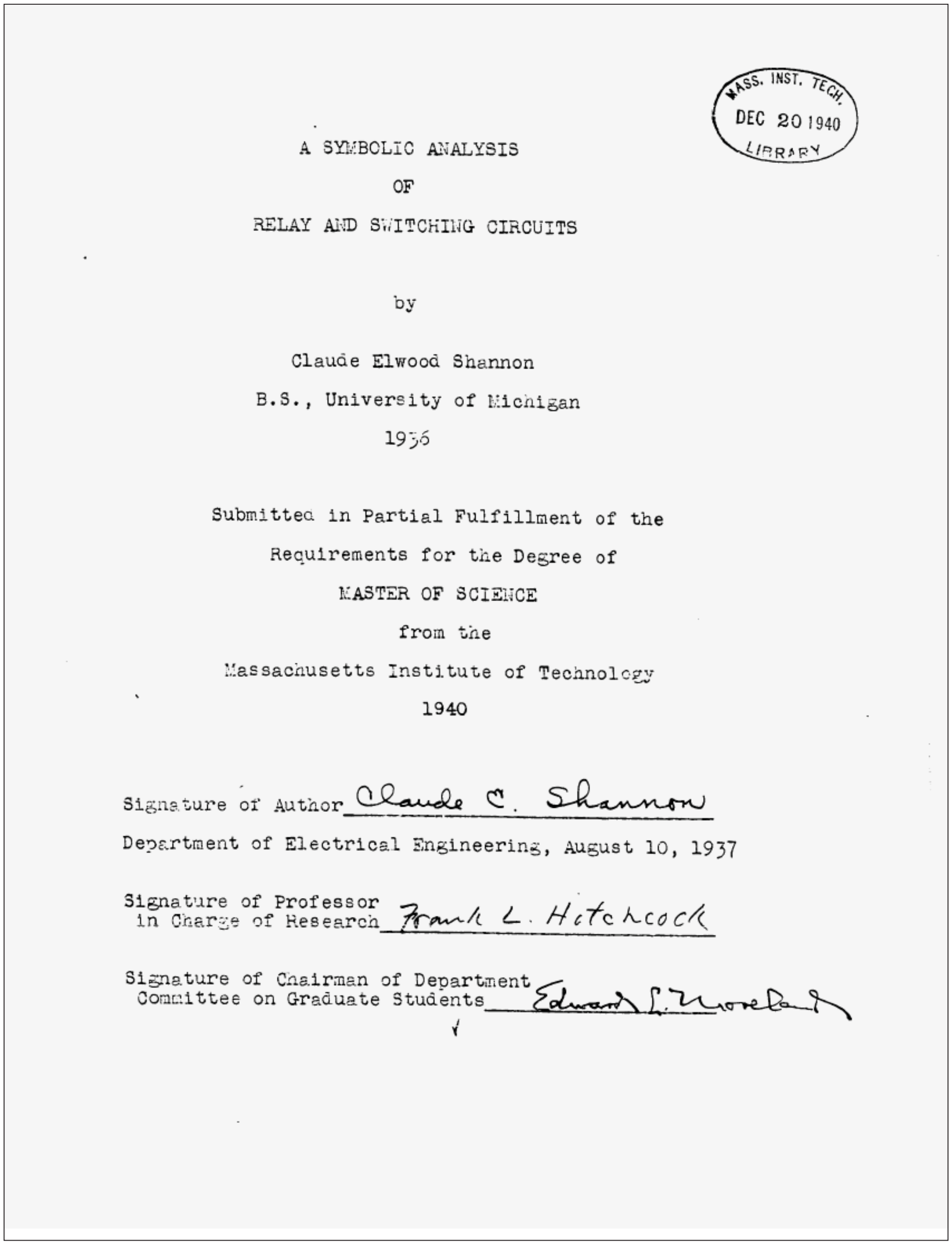
6. L.E. Dickson, *History of the Theory of Numbers*, Vol. I, Carnegie Institution of Washington, Chap. XIII.

This thesis is estimated by some scholars as the most frequently referenced master thesis of the 20th century. The main contributions were published in two related papers by C.E. Shannon [26], [27], see Fig. 2, Fig. 3, and Fig. 4. In these publications, Shannon used logic expressions in Boolean algebra to describe and simplify logic networks. Further publications by Shannon in these areas include [23], [24], [28].

In [27], there are 11 references including [23], [26], the book by L.Coutura (item 2 above), and the following references presented here again in the same formulation as in the original paper by Shannon

1. A. Nakashima, Various papers in *Nippon Electrical Communication Engineering*, April, Sept., Nov., Dec., 1938.

2. H. Piesch, Papers in *Archiv. fur Electrotechnik*, xxxiii, page 692 and page 733, 1939.



A SYMBOLIC ANALYSIS

OF

RELAY AND SWITCHING CIRCUITS

by

Claude Elwood Shannon

B.S., University of Michigan

1936

Submitted in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

from the

Massachusetts Institute of Technology

1940

Signature of Author Claude E. Shannon

Department of Electrical Engineering, August 10, 1937

Signature of Professor
in Charge of Research Frank L. Hitchcock

Signature of Chairman of Department
Committee on Graduate Students Edward L. Moreland

Figure 1: The first page of the MSc. thesis by C.E. Shannon in 1938.

A Symbolic Analysis of Relay and Switching Circuits*

*Claude E. Shannon***

I. Introduction

In the control and protective circuits of complex electrical systems it is frequently necessary to make intricate interconnections of relay contacts and switches. Examples of these circuits occur in automatic telephone exchanges, industrial motor-control equipment, and in almost any circuits designed to perform complex operations automatically. In this paper a mathematical analysis of certain of the properties of such networks will be made. Particular attention will be given to the problem of network synthesis. Given certain characteristics, it is required to find a circuit incorporating these characteristics. The solution of this type of problem is not unique and methods of finding those particular circuits requiring the least number of relay contacts and switch blades will be studied. Methods will also be described for finding any number of circuits equivalent to a given circuit in all operating characteristics. It will be shown that several of the well-known theorems on impedance networks have roughly analogous theorems in relay circuits. Notable among these are the delta-wye and star-mesh transformations, and the duality theorem.

The method of attack on these problems may be described briefly as follows: any circuit is represented by a set of equations, the terms of the equations corresponding to the various relays and switches in the circuit. A calculus is developed for manipulating these equations by simple mathematical processes, most of which are similar to ordinary algebraic algorisms. This calculus is shown to be exactly analogous to the calculus of propositions used in the symbolic study of logic. For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit. The circuit may then be immediately drawn from the equations. By this method it is always possible to find the simplest circuit containing only series and parallel connections, and in some cases the simplest circuit containing any type of connection.

Our notation is taken chiefly from symbolic logic. Of the many systems in common use we have chosen the one which seems simplest and most suggestive for our interpretation. Some of our phraseology, such as node, mesh, delta, wye, etc., is borrowed from ordinary network theory for simple concepts in switching circuits.

* *Transactions American Institute of Electrical Engineers*, vol. 57, 1938. (Paper number **38-80**, recommended by the AIEE committees on communication and basic sciences and presented at the AIEE summer convention, Washington, D.C., June 20-24, 1938. Manuscript submitted March 1, 1938; made available for preprinting May 27, 1938.)

** Claude E. Shannon is a research assistant in the department of electrical engineering at Massachusetts Institute of Technology, Cambridge. This paper is an abstract of a thesis presented at MIT for the degree of master of science. The author is indebted to Doctor F. L. Hitchcock, Doctor Vannevar Bush, and Doctor S. H. Caldwell, all of MIT, for helpful encouragement and criticism.

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Figure 2: The first page of the paper by Shannon [26].

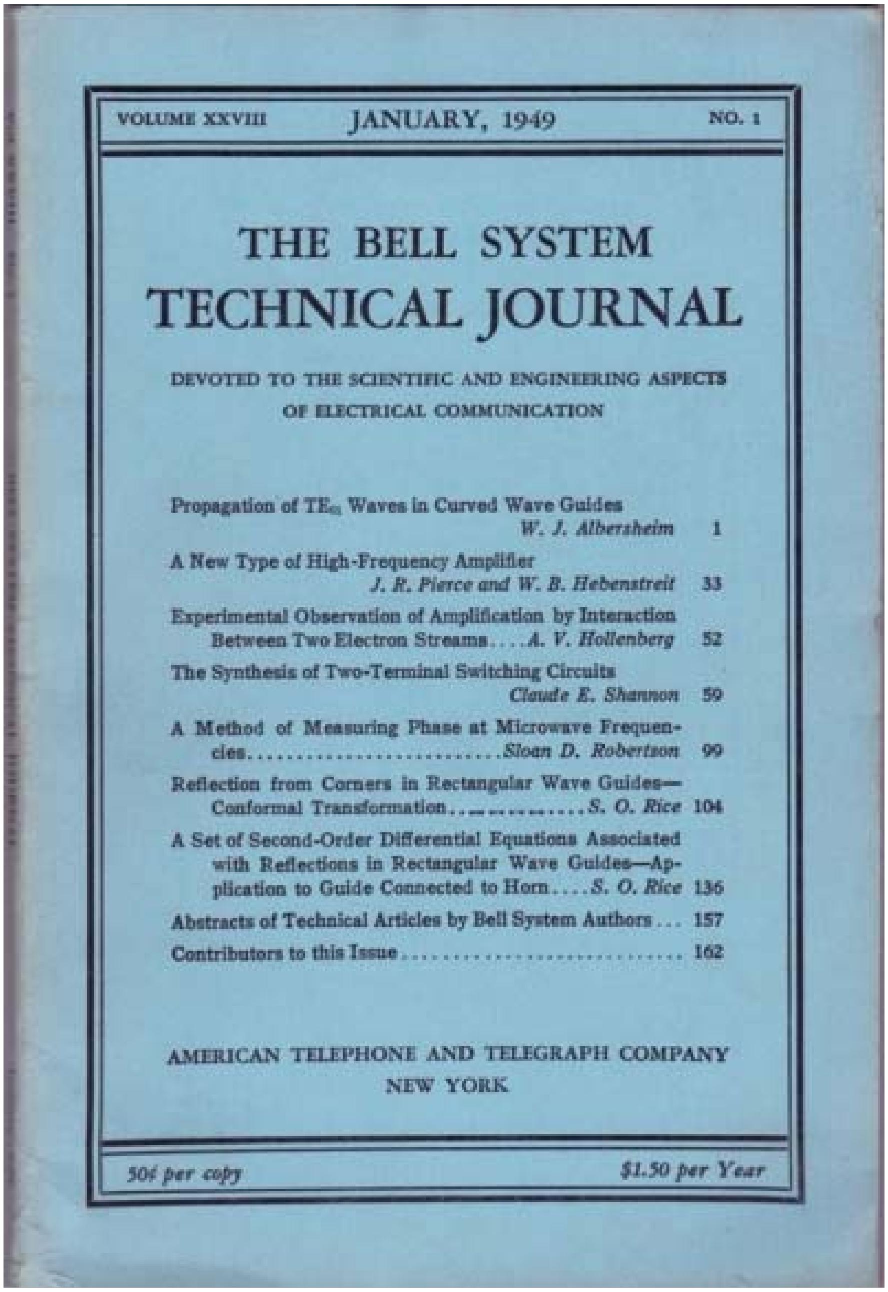

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Figure 3: The cover page of the *Bell System J.* where the paper [27] is published.

The Synthesis of Two-Terminal Switching Circuits

By CLAUDE. E. SHANNON

PART I: GENERAL THEORY

1. Introduction

THE theory of switching circuits may be divided into two major divisions, analysis and synthesis. The problem of analysis, determining the manner of operation of a given switching circuit, is comparatively simple. The inverse problem of finding a circuit satisfying certain given operating conditions, and in particular the *best* circuit is, in general, more difficult and more important from the practical standpoint. A basic part of the general synthesis problem is the design of a two-terminal network with given operating characteristics, and we shall consider some aspects of this problem.

Switching circuits can be studied by means of Boolean Algebra.[1,2] This is a branch of mathematics that was first investigated by George Boole in connection with the study of logic, and has since been applied in various other fields, such as an axiomatic formulation of Biology,[3] the study of neural networks in the nervous system,[4] the analysis of insurance policies,[5] probability and set theory, etc.

Perhaps the simplest interpretation of Boolean Algebra and the one closest to the application to switching circuits is in terms of propositions. A letter X, say, in the algebra corresponds to a logical proposition. The sum of two letters $X + Y$ represents the proposition "X or Y" and the product XY represents the proposition "X and Y". The symbol X' is used to represent the negation of proposition X, i.e. the proposition "not X". The constants 1 and 0 represent truth and falsity respectively. Thus $X + Y = 1$ means X or Y is true, while $X + YZ' = 0$ means X or (Y and the contradiction of Z) is false.

The interpretation of Boolean Algebra in terms of switching circuits[6,8,9,10] is very similar. The symbol X in the algebra is interpreted to mean a make (front) contact on a relay or switch. The negation of X, written X', represents a break (back) contact on the relay or switch. The constants 0 and 1 represent closed and open circuits respectively and the combining operations of addition and multiplication correspond to series and parallel connections of the switching elements involved. These conventions are shown in Fig. 1. With this identification it is possible to write an algebraic

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Figure 4: The first page of the paper by Shannon [27].

3. G.A. Mongomerie, "Sketch for an algebra of relay and contactor circuits", *J. I. of E.E.*, Vol. 9, Part 3, No. 36, July 1948, page 33.

In Japan, the same problem was studied by Akira Nakashima who published in 1935 his research results in [14], [15], [16] (Fig. 5, Fig. 6). Nakashima had an approach opposite to that used by Shannon, he analyzed a large number of different relay networks and devised an underlying theory. With the help of his associate, Masao Hanzawa, Nakashima formulated a theory that can be viewed as a kernel of Switching Theory. In the first seven condensed English translations of his papers, Nakashima does not provide references, except the second paper [18] where there is a reference to his first paper [14]. In 1941, Nakashima and Hanzawa [19], realized the relationship and strong coincidence of their theory with the work of G.J. Boole and E. Schröder and put references to their work [4], [25]

1. G. Boole, *An Investigation of the Laws of Thought*, London, 1854.
2. E. Schroder, *Vorlesunden uber die Algebra of Logic*, Band 1, 1890.

In [21], there is the reference to the work of B.A. Bernstein as follows

B.A. Bernstein, "Postulate for Boolean algebra involving the operation of complete disjunction", *Annals of Mathematics*, April 1936.

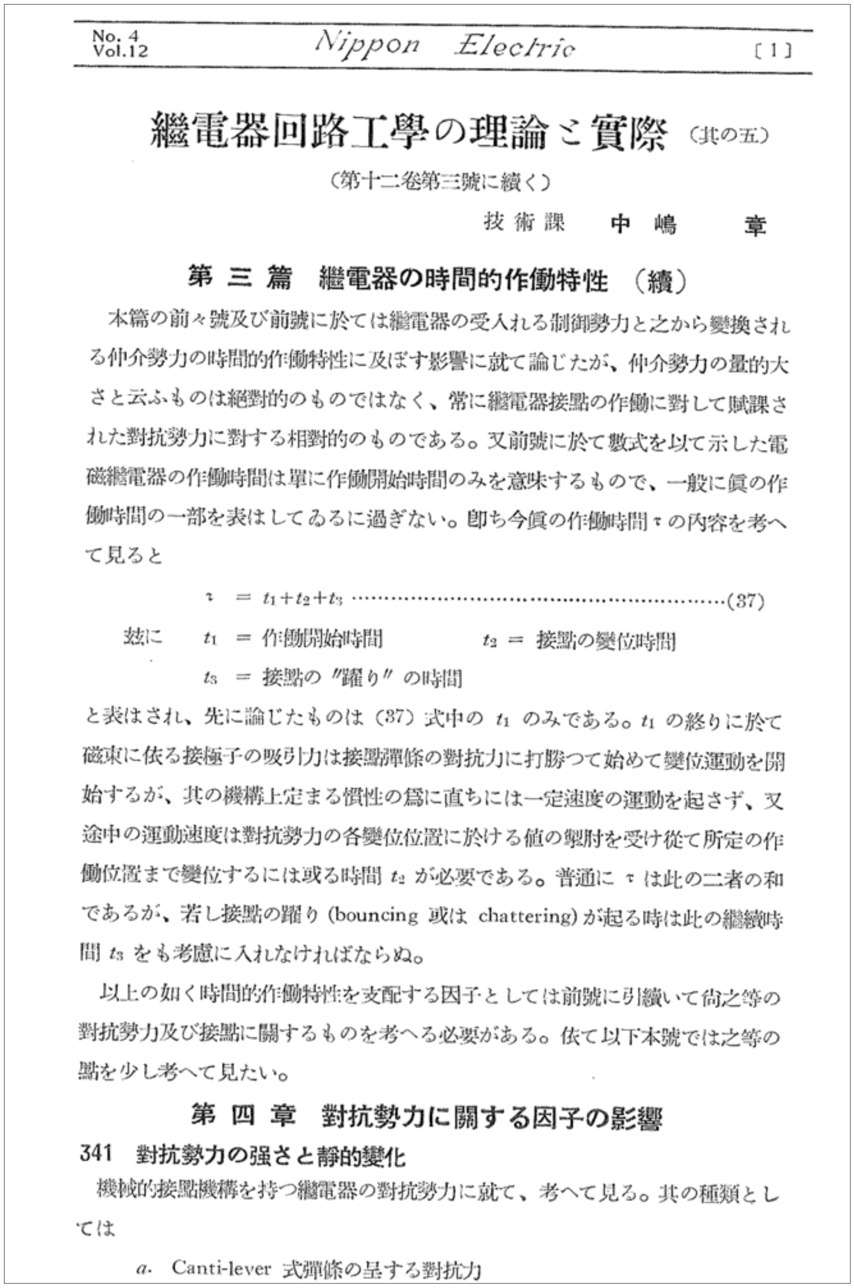
The following paper in Japanese,

Nakashima, A., "Theory of relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 220, March 1941, 9-12.

for which there is no English translation, is a short tutorial in which Nakashima presented basic postulates and theorems in the Boolean algebra (Fig. 7).

The first reference in this paper is in Japanese, and other three are

1. Boole, G., *An Investigation of the Laws of Thought*, London, 1854.
2. Schröder, E., *Vorlesungen über die Algebra der Logik*, 1890.
3. Couturat, L., *The Algebra of Logic*, 1914.



繼電器回路工學の理論と實際 (其の五)

(第十二卷第三號に續く)

技術課 中嶋 章

第三篇 繼電器の時間的作働特性 (續)

本篇の前々號及び前號に於ては繼電器の受入れる制御勢力と之から變換される仲介勢力の時間的作働特性に及ぼす影響に就て論じたが、仲介勢力の量的大さと云ふものは絕對的のものではなく、常に繼電器接點の作働に對して賦課された對抗勢力に對する相對的のものである。又前號に於て數式を以て示した電磁繼電器の作働時間は單に作働開始時間のみを意味するもので、一般に眞の作働時間の一部を表はしてゐるに過ぎない。卽ち今眞の作働時間 τ の內容を考へて見ると

$$\tau = t_1 + t_2 + t_3 \cdots\cdots(37)$$

茲に t_1 = 作働開始時間　t_2 = 接點の變位時間

t_3 = 接點の "躍り" の時間

と表はされ、先に論じたものは (37) 式中の t_1 のみである。t_1 の終りに於て磁束に依る接極子の吸引力は接點彈條の對抗力に打勝つて始めて變位運動を開始するが、其の機構上定まる慣性の爲に直ちには一定速度の運動を起さず、又途中の運動速度は對抗勢力の各變位位置に於ける値の掣肘を受け從て所定の作働位置まで變位するには或る時間 t_2 が必要である。普通に τ は此の二者の和であるが、若し接點の躍り (bouncing 或は chattering) が起る時は此の繼續時間 t_3 をも考慮に入れなければならぬ。

以上の如く時間的作働特性を支配する因子としては前號に引續いて尙之等の對抗勢力及び接點に關するものを考へる必要がある。依て以下本號では之等の點を少し考へて見たい。

第四章 對抗勢力に關する因子の影響

341 對抗勢力の强さと靜的變化

機械的接點機構を持つ繼電器の對抗勢力に就て、考へて見る。其の種類としては

a. Canti-lever 式彈條の呈する對抗力

Figure 5: The first page of the paper by Akira Nakashima in **Nichiden Geppo** Vol. 12, No. 4, April 1935, 1-13.

(29) The Theory of Relay Circuit Composition 197

THE THEORY OF RELAY CIRCUIT COMPOSITION

Akira Nakashima, Member

(Nippon Electric Co., Ltd. Tokyo)

CONTENT.

I. General Essence of Relay Circuit.
II. On Action Element.
III. On Contact Points.
IV. Considerations Regarding Simple Partial Path.
V. Considerations Regarding Complex Partial Path.
VI. Considerations Regarding Energy Transmitting Path.
VII. Time Action Forms of Relay and Their Objects.
VIII. Some of Fundamental Types of Relay Circuit.
IX. Conclusion.

SYNOPSIS.

This is a general discussion of and a systematic consideration on the composition of so-called relay circuit system, which has made surprising advancement in recent time in connection with automatic telephone exchange, remote control systems, etc.

It shows the fundamental idea and characteristics of the relay circuit, some of the interesting properties and theorems regarding the dynamic geometrical character, analytical treatments of simple cases, forms of relays, and then some of the fundamental systems of relay circuit composition.

It is noted that transient phenomena which arise inevitably in the relay circuit are not, however, included in this discussion.

I. GENERAL ESSENCE OF RELAY CIRCUIT.

1.1. Fundamental idea and definitions.

The relay circuits now in use are many in kinds and complex in variation. However, under general survey, the following definitions may briefly be given:

Relay Circuit is a method in which it becomes a mediator between some given phenomena and the corresponding desired phenomena, and by the use of relays as its composite elements, the occurrence of the former realizes the latter.

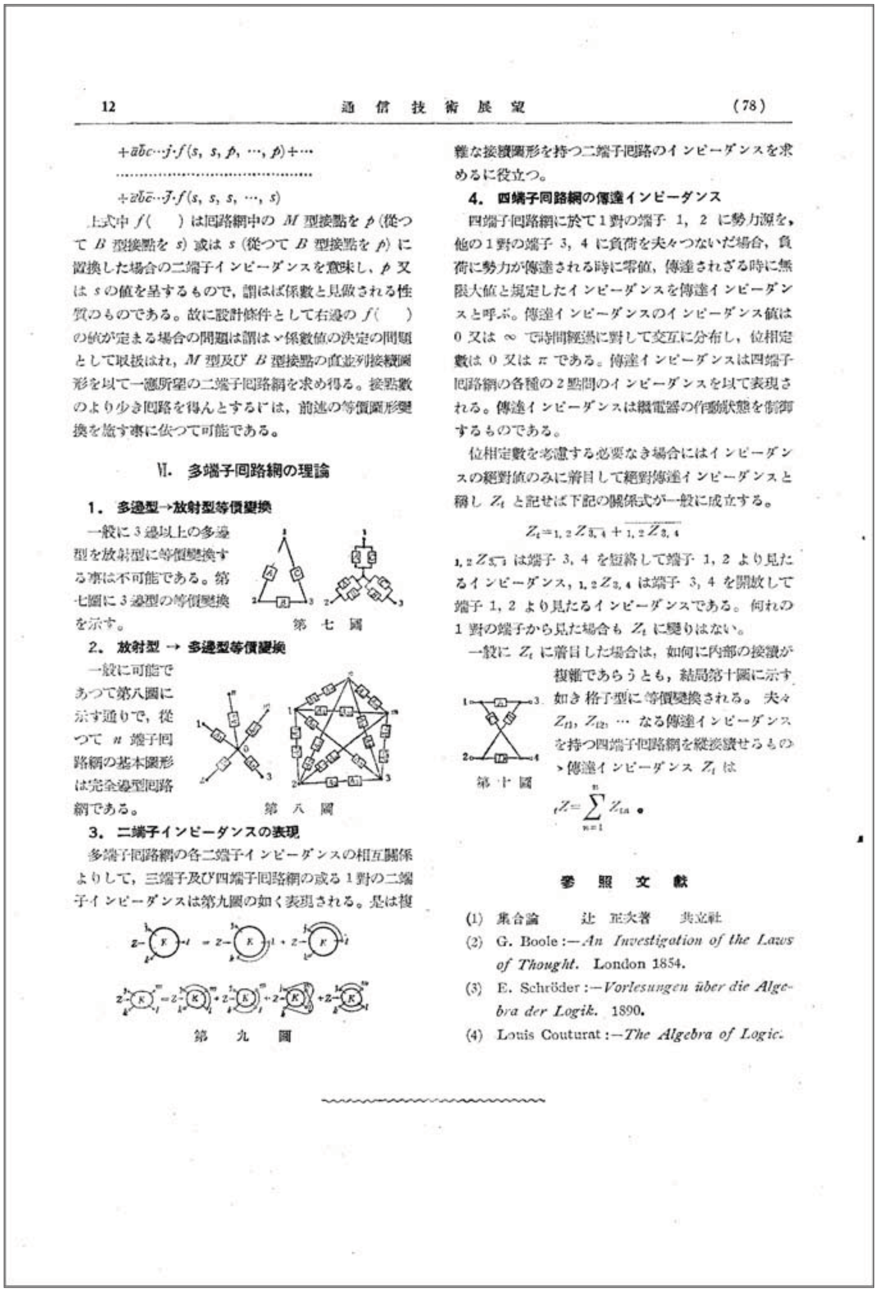
Next, taking these relays as its composite element in broad sense:

The relays may be defined as an element that determines, by presence or absence of its receiving energy, whether another energy is transmitted or not. In regard to energy the former is called controlling and the latter controlled energy. They are, however, termed merely with respect to one certain relay; and one energy may become sometimes the former and other times the latter. Thus, from this definition we find that the relay consists, in one part, of receiving controlling energy to determine its action, and in another part, of controlling directly the transmission of controlled energy. The former may be called acting element and the latter contact point. Contact point may be said as a composite element of transmitting path of controlled energy, which is controlled at its point. It is also made of a number of contact elements that have definite meaning in regard to mechanical contact point.

Accordingly, in the relay circuit, the energy transmitting path is made in general of energy source, acting elements, contact points, and of path element which is a part of the path that connects all of these. The path element containing no impedance against controlled energy is called simple

J. I. T. T. E. of Japan, No. 150, September 1935.

Figure 6: The first page of the English version of the paper [14].



12　　　　通 信 技 術 展 望　　　　(78)

$$+\bar{a}\bar{b}c\cdots j\cdot f(s,\ s,\ p,\ \cdots,\ p)+\cdots$$

……………………………………

$$+\bar{a}\bar{b}\bar{c}\cdots\bar{j}\cdot f(s,\ s,\ s,\ \cdots,\ s)$$

上式中 $f(\quad)$ は回路網中の M 型接點を p (從つて B 型接點を s) 或は s (從つて B 型接點を p) に置換した場合の二端子インピーダンスを意味し、p 又は s の値を呈するもので，謂はば係數と見做される性質のものである。故に設計條件として右邊の $f(\quad)$ の値が定まる場合の問題は謂はゞ係數値の決定の問題として取扱はれ，M 型及び B 型接點の直並列接續圖形を以て一應所望の二端子回路網を求め得る。接點數のより少き回路を得んとするには，前述の等價圖形變換を施す事に依つて可能である。

Ⅵ. 多端子回路網の理論

1. 多邊型→放射型等價變換

一般に 3 邊以上の多邊型を放射型に等價變換する事は不可能である。第七圖に 3 邊型の等價變換を示す。

第 七 圖

2. 放射型 → 多邊型等價變換

一般に可能であつて第八圖に示す通りで，從つて n 端子回路網の基本圖形は完全邊型回路網である。

第 八 圖

3. 二端子インピーダンスの表現

多端子回路網の各二端子インピーダンスの相互關係よりして，三端子及び四端子回路網の或る 1 對の二端子インピーダンスは第九圖の如く表現される。是は複雜な接續圖形を持つ二端子回路のインピーダンスを求めるに役立つ。

第 九 圖

4. 四端子回路網の傳達インピーダンス

四端子回路網に於て 1 對の端子 1, 2 に勢力源を，他の 1 對の端子 3, 4 に負荷を夫々つないだ場合，負荷に勢力が傳達される時に零値，傳達されざる時に無限大値と規定したインピーダンスを傳達インピーダンスと呼ぶ。傳達インピーダンスのインピーダンス値は 0 又は ∞ で時間經過に對して交互に分布し，位相定數は 0 又は π である。傳達インピーダンスは四端子回路網の各種の 2 點間のインピーダンスを以て表現される。傳達インピーダンスは繼電器の作動狀態を制御するものである。

位相定數を考慮する必要なき場合にはインピーダンスの絕對値のみに着目して絕對傳達インピーダンスと稱し Z_t と記せば下記の關係式が一般に成立する。

$$Z_t={}_{1,2}Z_{\overline{3,4}}+\overline{{}_{1,2}Z_{3,4}}$$

${}_{1,2}Z_{\overline{3,4}}$ は端子 3, 4 を短絡して端子 1, 2 より見たるインピーダンス，${}_{1,2}Z_{3,4}$ は端子 3, 4 を開放して端子 1, 2 より見たるインピーダンスである。何れの 1 對の端子から見た場合も Z_t に變りはない。

一般に Z_t に着目した場合は，如何に内部の接續が複雜であらうとも，結局第十圖に示す如き格子型に等價變換される。夫々 Z_{t1}, Z_{t2}, … なる傳達インピーダンスを持つ四端子回路網を縱接續せるものゝ傳達インピーダンス Z_t は

第 十 圖

$$Z_t=\sum_{n=1}^{n}Z_{tn}\ .$$

參 照 文 獻

(1) 集合論　　辻 正次著　　共立社

(2) G. Boole :—*An Investigation of the Laws of Thought.* London 1854.

(3) E. Schröder :—*Vorlesungen über die Algebra der Logik.* 1890.

(4) Louis Couturat :—*The Algebra of Logic.*

Figure 7: The page 4 of the paper by Nakashima published in March 1941.

The paper

Nakashima, A., "Theory of relay circuit", *Journal of the Institute of Electrical Communication Engineers of Japan*, No. 220, July 1941, 397-406,

is the speech delivered by Akira Nakashima at the general assembly of the IECEJ on 26th April 1941. It covers his major research results. At the third page of this paper, there is a table with basic postulates and theorems in the Boolean algebra. Besides references as in the above paper, Nakashima has mentioned the Journal of Symbolic Logic, 1936, and the book by Couturat (Fig. 8).

For more details on the work of Nakashima and a list of publications, see [32], [34], [35].

In The Soviet Union, early research on this subject was also done in the late thirties, resulting in a PhD thesis in the physic-mathematical sciences by Viktor Ivanovič Šestakov, defended on September 28, 1938, at the State University Lomonosov, Moscow, Soviet Union [29]. In the thesis, Šestakov referred to the work on logic by Glivenko [9], and Žegalkin and Sludskaja [42]. The major part of the thesis of Šestakov was published in [30], [31] (Fig. 9). For discussions on the work by Šestakov, see [2], [3], [7], [11].

For historical accuracy, it should be noticed that the first remark on the applicability of the algebra of logic, whose central part is Boolean algebra, in logic network design, is due to the physicist Paul Ehrenfest as early as 1910. These remarks are presented in a review of the book *Algebra of Logic* by Louis Couturat (Table 1). The review was published in *Žurnal Russkago Fiziko-hemičeskago Obščestva*, Fizičeskij otdel (Journal of the Russian Physical-Chemistry Society), Part for Physics, Vol. 42, 1910, Second part, 382-387 [8].

The first report of this review by Ehrenfest in the western literature is ascribed to G.L. Kline [12] who pointed it out in 1951 in the review of a paper by S.A. Anovskaja [1].

Related remarks on the work of Ehrenfest were reported in an article by G.N. Povarov that was mentioned in 1959 in a review by Comey and Kline of a paper by Zinoviev [6]. Also, in a review by A. Church of a paper by T.A. Kalin [5], the same fact is pointed out with a statement that the author of the review had not seen the paper by Ehrenfest and had been informed about it by G.L. Kline.

In 1966, in [11], the following was stated

A 1910 book review by P. Ehrenfest [8] is sometimes mentioned as the first recognition that the algebra of logic might be used as an analytical tool

唯 + と ×，直並列の關係だけしか現はせません。

7. 將來の研究問題

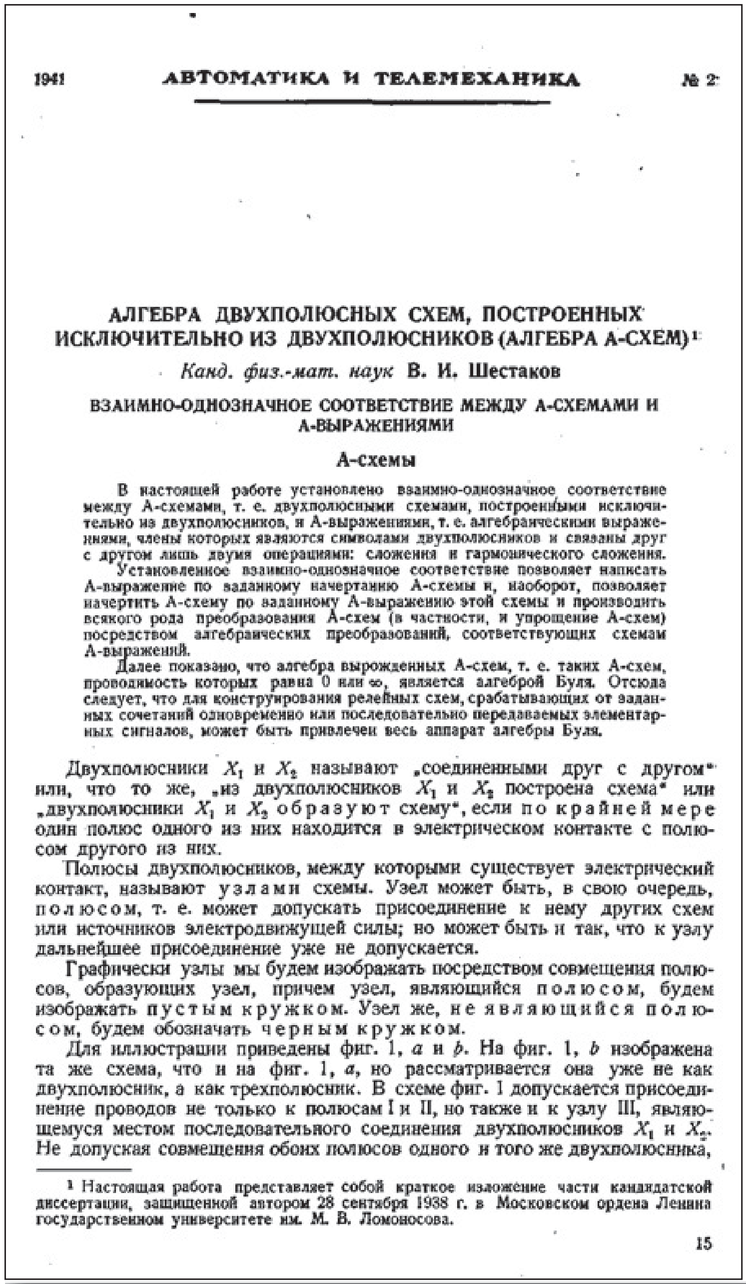
尙ほ今後吾々が考へなければならない問題と致しましては色々あると思ひます。實は私共のやつて居りますのは今御說明申上げたやうな內容の範圍に止まつて居るのでありまして，更に今後皆樣の御協力を得まして硏究致したいと思ひますのは，能動網には入つて行かなければならんといふことが一つであります。本當の目的は能動網にまで入ることにあるのでございまして，能動網の方までも理論を展開する必要がある，さうしないと實際問題としまして問題を處理するにはまだまだ距離がある。この能動網の硏究を至急に必要とする狀態であります。それから，さういふ能動網の問題を處理しますためには勢ひ Boole 代數に於ける方程式論といふものが必要になるのでありますが，殘念ながら方程式論は極く局限された範圍だけしか現在の所では解けないやうであります。と申しますのは，初等代數と違ひ極めて特殊な基礎性質を持つて居りますために一意的な解といふものが引出しにくいのであります。併しどう致しましてもこの方程式論の方は今後もつと突ついて見なければいかんのであります。或る程度のものは旣に雜誌に發表させて戴きましたが，あんなのでは迚も駄目でありまして，もつと深くこれを突ついて行かなければ能動網の實際問題を解くことは難かしいと思ふのであります。

もう一つの問題は，接續圖形の表現法として + 及び × 記號による直並列接續だけに限定されない，もつとうまい表現法はないか，例へばマトリックスの形といふやうなものを使ひましてもつと普遍的な接續圖形の取扱ひ方はないかといふ問題であります。それに就きましては東北遞信硏究所の岡田さんが相當突込んで硏究して居られると存じますが，この方面は是非とも皆樣の御協力を得ましてやつて行かなければいかんと思ふのであります。ところが，まだ現在の所私の知る範圍に於きましては殘念ながらブール演算に於けるマトリックスといふものを餘り發表された例を聞かないのでありまして，一二の文獻もありますけれども殆ど役に立たないやうに思つて居ります。

最後に斯ういふ問題をやつて行くのに何か御參考になるかと思ひまして，少しばかり參照すべき文獻を申上げて置きたいと思ふのであります。

ブール代數と申しますのは，最近數學の部門で相當活潑に論議されて居るものゝ一部でございまして，それに關聯しましては色々な本が出て居るやうに聞いて居ります。例へば北海道帝大から出して居る位相數學といふものは年に四回出す豫定だと聞いて居りますし，それから集合論に就きましては辻さんの集合論或は黑田さんの數學基礎論もございますが，それ以外に歷史的なものを申しますと George Boole 所謂ブール代數といふ名前の起りである人の An Investigation of the laws of thought (1854). これは隨分古いものでございますが論理數學の濫觴としまして參考になることと思ひます。次に E. Schröder の Vorlsungen über die Algebra der Logik (1890) これは相當分厚い本でございまして，三冊から成つて居りますが，これは詳細にドイツ式に論理數學を突ついて居ります。それから Louis Couturat の The Algebra of Logic (1914). これはフランス版を飜譯したものでありますが，これは極く手頃にブール代數といふものをよく取纏めたものと存じます。問題を處理するためには丁度恰好なものではないかと思つて居ります。それ以外にアメリカで出して居ります所の Journal of the Symbolic Logic. これには論理數學の文獻も相當出て居ります。例へば 1936 年までのものは一つに纏めてありますし其の後の文獻もずつと毎年出て居ります。これには相當ブール代數關係の論文が出て居りますので役立つと思ひます。それ以外にアメリカでは相當論理數學が發展して居りまして，アメリカの數學學會雜誌には時々面白い論文が出て居ると思ひます。實は私は數學をよく存じませんものですから，數學の專門の方に伺ひたいと常々思つて居りますが，直ちに取つて以て繼電器回路の方に役立つといふやうなペーパーは割合に少いのでありまして，斯ういふ文獻に絕えず注意して置く必要があるのではないかと思ふのでございます。長々と御淸聽を煩はしましたがこれで私のお話を終ります。

Figure 8: The page 10 of the paper by Nakashima in July 1941.



1941 АВТОМАТИКА И ТЕЛЕМЕХАНИКА № 2

АЛГЕБРА ДВУХПОЛЮСНЫХ СХЕМ, ПОСТРОЕННЫХ ИСКЛЮЧИТЕЛЬНО ИЗ ДВУХПОЛЮСНИКОВ (АЛГЕБРА А-СХЕМ)[1]

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ВЗАИМНО-ОДНОЗНАЧНОЕ СООТВЕТСТВИЕ МЕЖДУ А-СХЕМАМИ И А-ВЫРАЖЕНИЯМИ

А-схемы

В настоящей работе установлено взаимно-однозначное соответствие между А-схемами, т. е. двухполюсными схемами, построенными исключительно из двухполюсников, и А-выражениями, т. е. алгебраическими выражениями, члены которых являются символами двухполюсников и связаны друг с другом лишь двумя операциями: сложения и гармонического сложения.

Установленное взаимно-однозначное соответствие позволяет написать А-выражение по заданному начертанию А-схемы и, наоборот, позволяет начертить А-схему по заданному А-выражению этой схемы и производить всякого рода преобразования А-схем (в частности, и упрощение А-схем) посредством алгебраических преобразований, соответствующих схемам А-выражений.

Далее показано, что алгебра вырожденных А-схем, т. е. таких А-схем, проводимость которых равна 0 или ∞, является алгеброй Буля. Отсюда следует, что для конструирования релейных схем, срабатывающих от заданных сочетаний одновременно или последовательно передаваемых элементарных сигналов, может быть привлечен весь аппарат алгебры Буля.

Двухполюсники X_1 и X_2 называют „соединенными друг с другом" или, что то же, „из двухполюсников X_1 и X_2 построена схема" или „двухполюсники X_1 и X_2 образуют схему", если по крайней мере один полюс одного из них находится в электрическом контакте с полюсом другого из них.

Полюсы двухполюсников, между которыми существует электрический контакт, называют узлами схемы. Узел может быть, в свою очередь, полюсом, т. е. может допускать присоединение к нему других схем или источников электродвижущей силы; но может быть и так, что к узлу дальнейшее присоединение уже не допускается.

Графически узлы мы будем изображать посредством совмещения полюсов, образующих узел, причем узел, являющийся полюсом, будем изображать пустым кружком. Узел же, не являющийся полюсом, будем обозначать черным кружком.

Для иллюстрации приведены фиг. 1, *a* и *b*. На фиг. 1, *b* изображена та же схема, что и на фиг. 1, *a*, но рассматривается она уже не как двухполюсник, а как трехполюсник. В схеме фиг. 1 допускается присоединение проводов не только к полюсам I и II, но также и к узлу III, являющемуся местом последовательного соединения двухполюсников X_1 и X_2. Не допуская совмещения обоих полюсов одного и того же двухполюсника,

[1] Настоящая работа представляет собой краткое изложение части кандидатской диссертации, защищенной автором 28 сентября 1938 г. в Московском ордена Ленина государственном университете им. М. В. Ломоносова.

15

Figure 9: The first page of the paper by V.I. Šestakov in **Avtomatika i Telemekhanika**, Vol. 2, No. 6, 1941, 15-24.

for telephone switching networks. Church [5] in a 1953 book review, also grants Ehrenfest this priority. Nakasima [17] is rarely mentioned in this connection.

The work of Ehrenfest was also reported by H. Zamanek in 1993 in [38], where it was stated

Paul Ehrenfest, the famous Austrian physicist and friend of Albert Einstein, had postulated switching algebra as logical algebra already in 1910 - but in Russian, in an unknown St. Petersburg journal of physics and chemistry, and in a book review (of Couturat's Logic). So his perfectly clear insight remained unknown [8].

In [38] a reference to an earlier publication of Zemanek on the same subject was given [39]. See also [36], [37], [40], [41].

References to the comments and translations of parts of the review of Ehrenfest written by Zemanek in German are given in [13].

To the best of our knowledge, except this part of the review by Ehrenfest that was translated into German by Heinz Zemanek [37], no translation into English or other languages was published. With this motivation, in this booklet, we reprint the review by Ehrenfest and provide a translation of it into English accompanied by a brief analysis and discussion of related references.

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Figure 10: Louis Couturat (photo taken from Wikipedia).

2 The Algebra of Logic by Louis Couturat

Table 1 shows the different editions of the book *The Algebra of Logic* by Louis Couturat (Fig. 10). The Russian edition of this book motivated Paul Ehrenfest to write a review of it, pointing out that the algebra of logic can be used as an underlying mathematical theory for the design of logic networks [8]. This review is reprinted and translated into English in Section 4.

The biography of Louis Couturat can be found in several publications. We refer to the probably most detailed among them

Claro C. Dassen, "Vida y Obra de Luis Couturat", *Anales de la Academia National de Ciencias Exactas, Fisicas, y Natruales de Buenos Aires*, Vol. 4, 1939, 73-204.

Table 1: Editions of the book by Louis Couturat.

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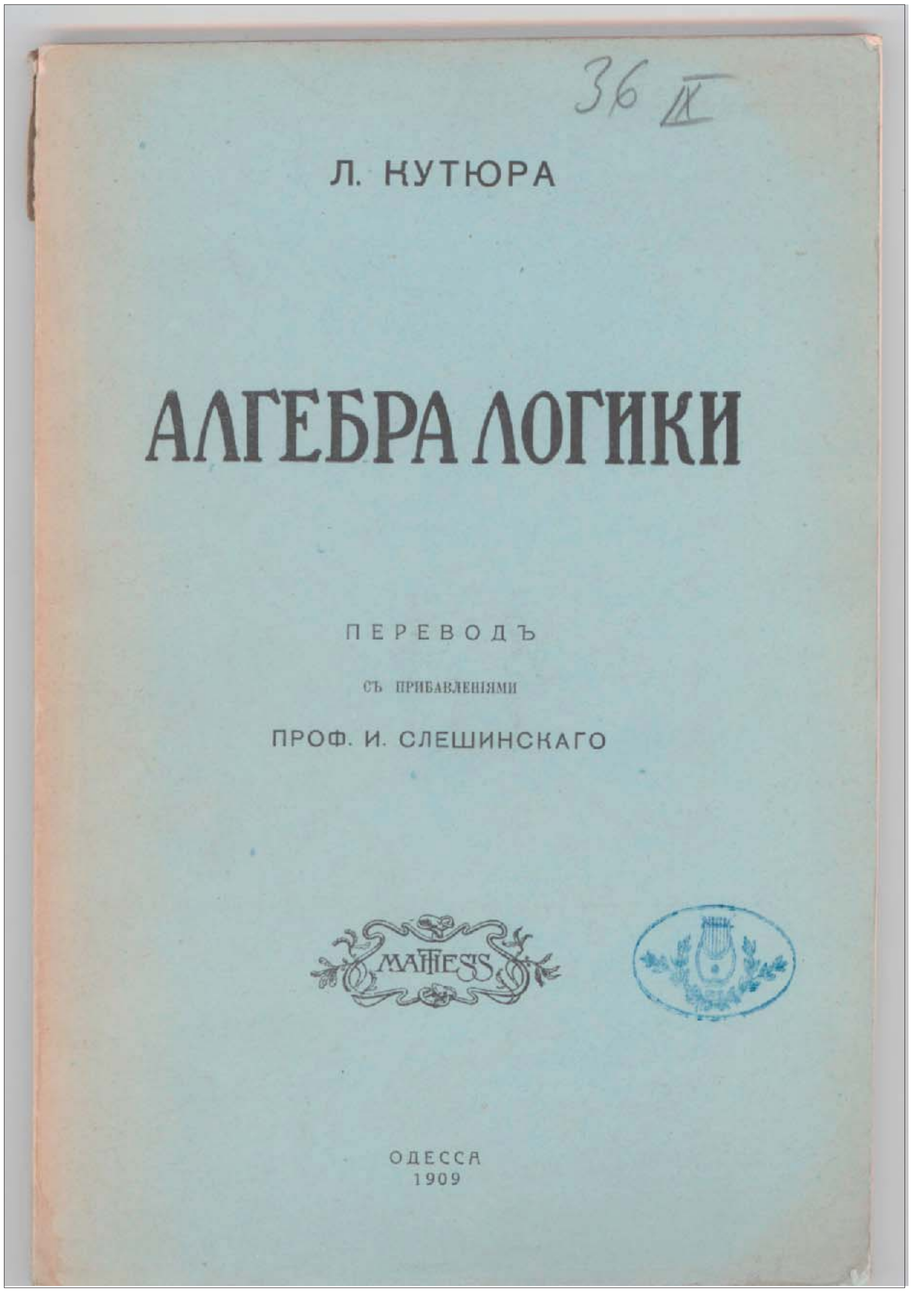
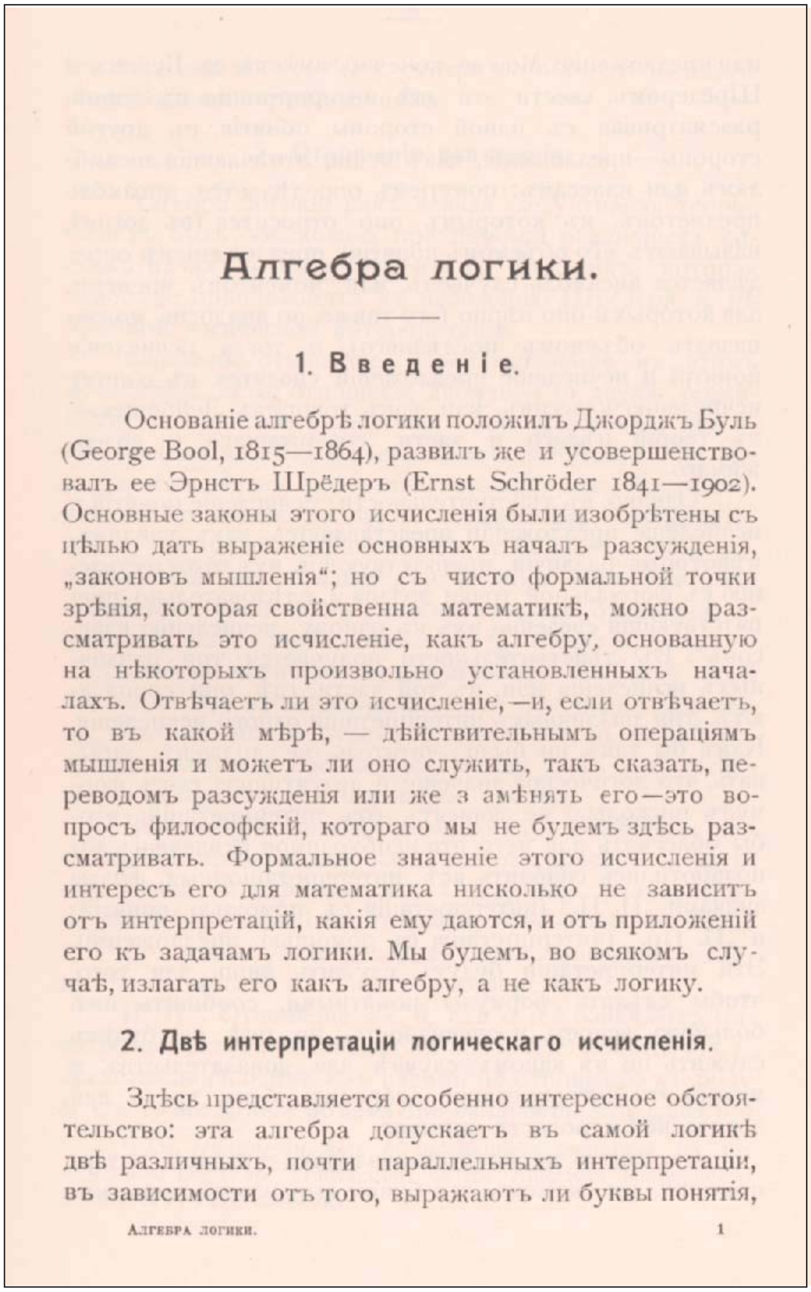


Figure 11: The cover of the Russian edition of the book by Louis Couturat.



Алгебра логики.

1. Введеніе.

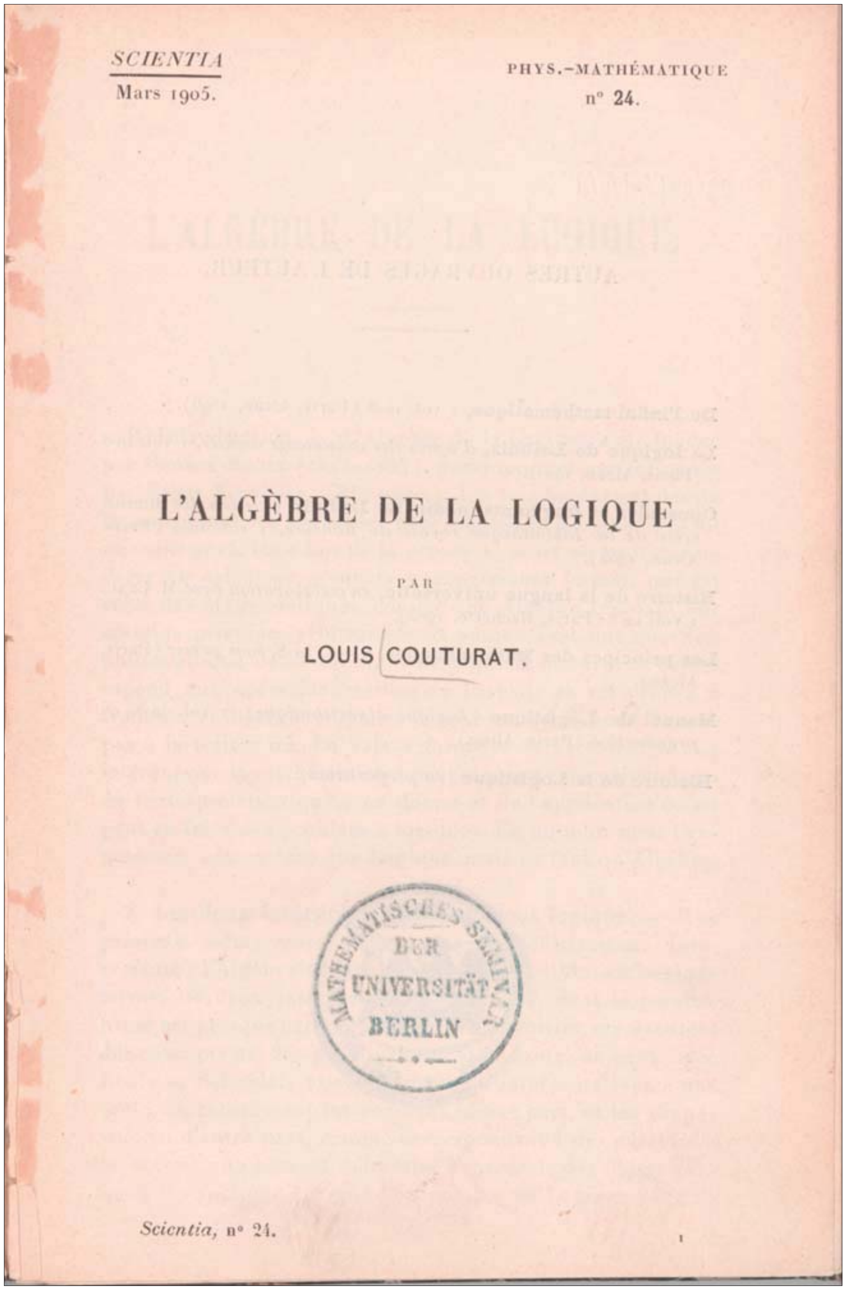
Основаніе алгебрѣ логики положилъ Джорджъ Буль (George Bool, 1815—1864), развилъ же и усовершенствовалъ ее Эрнстъ Шрёдеръ (Ernst Schröder 1841—1902). Основные законы этого исчисленія были изобрѣтены съ цѣлью дать выраженіе основныхъ началъ разсужденія, „законовъ мышленія"; но съ чисто формальной точки зрѣнія, которая свойственна математикѣ, можно разсматривать это исчисленіе, какъ алгебру, основанную на нѣкоторыхъ произвольно установленныхъ началахъ. Отвѣчаетъ ли это исчисленіе, —и, если отвѣчаетъ, то въ какой мѣрѣ, — дѣйствительнымъ операціямъ мышленія и можетъ ли оно служить, такъ сказать, переводомъ разсужденія или же з амѣнять его—это вопросъ философскій, котораго мы не будемъ здѣсь разсматривать. Формальное значеніе этого исчисленія и интересъ его для математика нисколько не зависитъ отъ интерпретацій, какія ему даются, и отъ приложеній его къ задачамъ логики. Мы будемъ, во всякомъ случаѣ, излагать его какъ алгебру, а не какъ логику.

2. Двѣ интерпретаціи логическаго исчисленія.

Здѣсь представляется особенно интересное обстоятельство: эта алгебра допускаетъ въ самой логикѣ двѣ различныхъ, почти параллельныхъ интерпретаціи, въ зависимости отъ того, выражаютъ ли буквы понятія,

АЛГЕБРА ЛОГИКИ. 1

Figure 12: The title page of the Russian edition of the book by Louis Couturat.



SCIENTIA
Mars 1905.

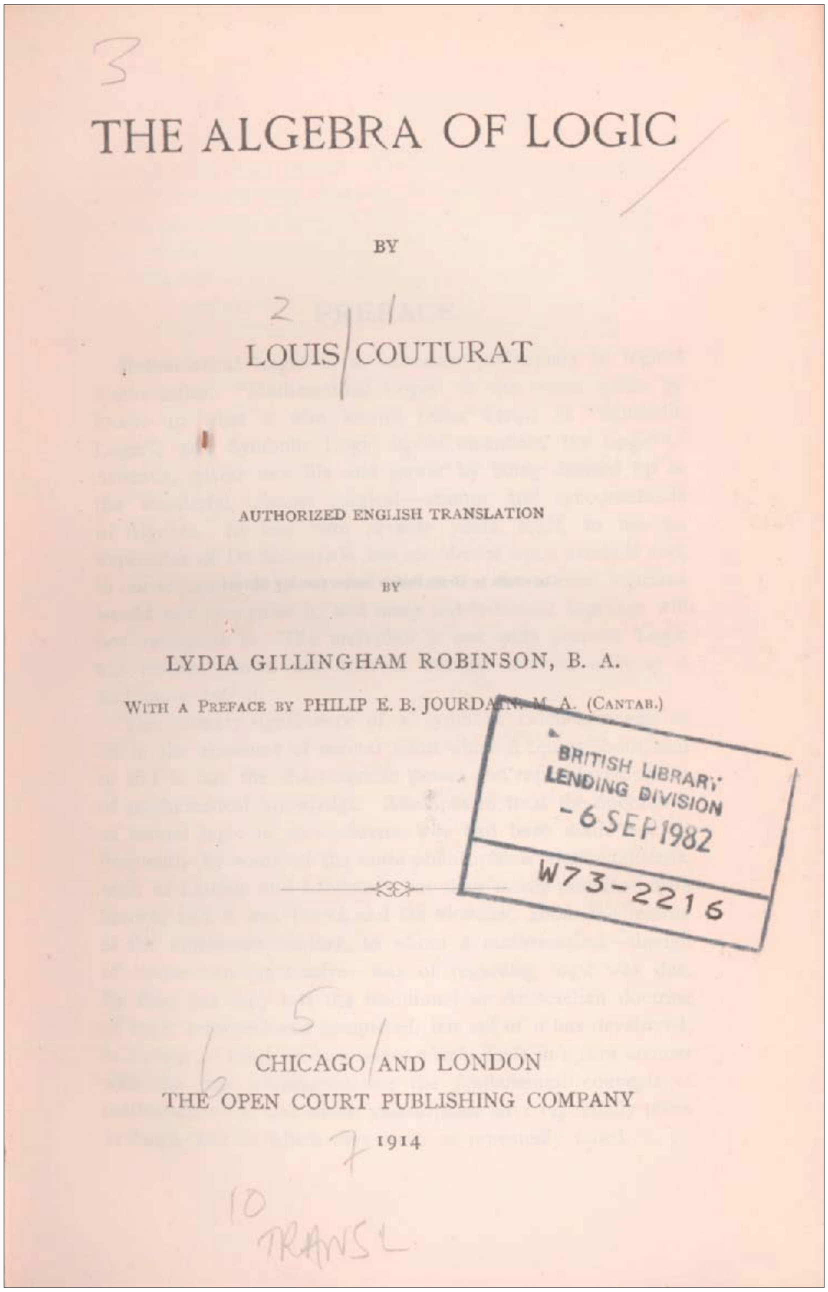
PHYS.-MATHÉMATIQUE
n° 24.

L'ALGÈBRE DE LA LOGIQUE

PAR

LOUIS COUTURAT.

Figure 13: The title page of the French edition of the book by Louis Couturat.


THE ALGEBRA OF LOGIC

BY

LOUIS COUTURAT

AUTHORIZED ENGLISH TRANSLATION

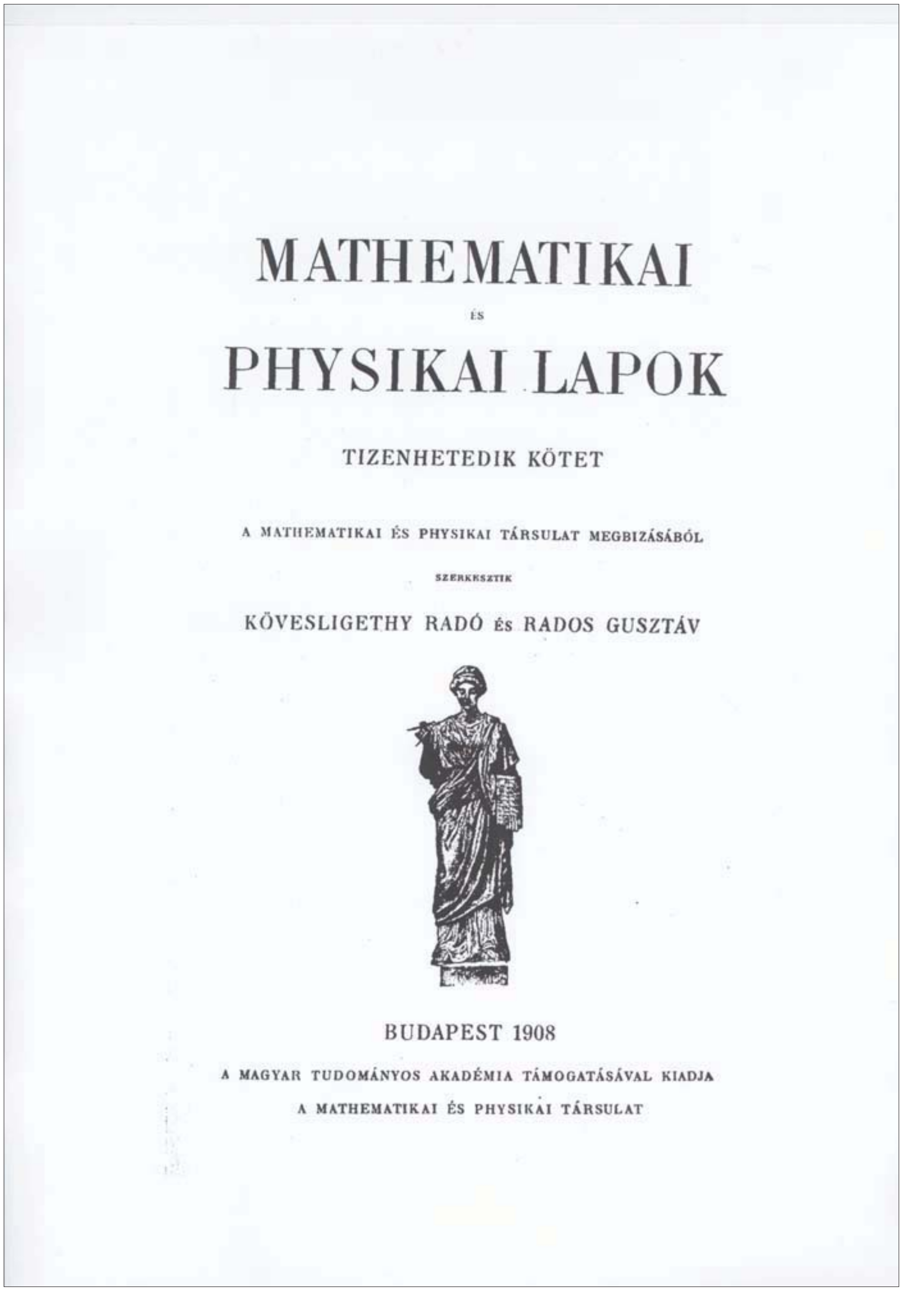
BY

LYDIA GILLINGHAM ROBINSON, B. A.

WITH A PREFACE BY PHILIP E. B. JOURDAIN, M. A. (CANTAB.)

CHICAGO AND LONDON
THE OPEN COURT PUBLISHING COMPANY
1914

Figure 14: The title page of the English edition of the book by Louis Couturat.



MATHEMATIKAI

ÉS

PHYSIKAI LAPOK

TIZENHETEDIK KÖTET

A MATHEMATIKAI ÉS PHYSIKAI TÁRSULAT MEGBIZÁSÁBÓL

SZERKESZTIK

KÖVESLIGETHY RADÓ ÉS RADOS GUSZTÁV

BUDAPEST 1908

A MAGYAR TUDOMÁNYOS AKADÉMIA TÁMOGATÁSÁVAL KIADJA

A MATHEMATIKAI ÉS PHYSIKAI TÁRSULAT

Figure 15: The title page of the Hungarian edition of the book by Louis Couturat.

Figure 16: Paul Ehrenfest (photo from Wikipedia).

3 Paul Ehrenfest

In the literature, the name of Paul Ehrenfest appears in different pronunciation as Ehrenfest, Erenfest, and Erénfést.

Paul Ehrenfest is a world renowned physicist whose main research interests were quantum theory, relativity theory, and statistical mechanics. For example, Ehrenfest is known for his work on the theory of phase transition of thermodynamic systems and for the Ehrenfest theorem in quantum mechanics. After publishing

Paul Ehrenfest, "Zur Planckschen Strahlungstheorie", *Physikalische Zeitschrift*, Vol. 7, 1906, 528532 reprinted in Collected Scientific Papers, M.J. Klein (ed.), North-Holland, 1959, 120-124.

Ehrenfest got a reputation for being among the first physicists to endorse the revolutionary theories of Albert Einstein with whom he later became a personal friend. Einstein appreciated Ehrenfest, particularly after he had heard a lecture that Ehrenfest gave at the German University in Prague in 1910.

Most of the biographers of Paul Ehrenfest point out two facts that considerably influenced both his personal life and professional work. Paul Ehrenfest was an Austrian citizen of Jewish origin. This fact, combined with another fact of a similar kind resulted to a specific state of mind, leading finally to Erenfest's tragic end by suicide. In order to marry an Orthodox Russian lady, Tatyana Alexeyevna Afanassjewa, a mathematician, both his wife and he had to declare themselves as nondenominational which was a way to avoid the rigid Austrian law regulations. At that time, such origins and religious backgrounds were not very helpful for finding a position at a university or good permanent employment as an engineer or scientist. The related difficulties and disappointments were the main characteristics of the first several years of the professional career of Paul Ehrenfest which can be summarized as follows.

Paul Ehrenfest received his Ph.D. degree at the University of Vienna in June 1904. Being jobless for two years, after unsuccessful attempts to find employment in Göttingen, Germany, where he and his wife were students, in the summer of 1907 they moved to St. Petersburg, Russia.

Thanks to his reputation as a well-known physicist, mainly due to his above mentioned paper from 1906, Ehrenfest established contacts with physicists in St. Petersburg and with the very famous mathematician Vladimir Andreevich Steklov. This, however, did not help him to get a permanent position at the University of St. Petersburg in spite of his efforts to establish the necessary links. The major obstacles were the above mentioned facts of his origins and personal life that were strongly opposed by Russian society at the time, as well as his criticism of the old-fashioned way of work at the university and the rigid study procedure.

Due to an invitation by Steklov, Ehrenfest gave several lectures at the University of St. Petersburg on different mathematical subjects. Ehrenfest also became a member of the editorial board of the *Journal of the Russian Physical-Chemical Society*, being especially engaged in publishing a supplement of this journal entitled *Problems in Physics*. While serving as a member of the editorial board, Ehrenfest regularly attended meetings of the Russian Physical-Chemical Society and published several articles and book reviews including the review translated and reprinted in this booklet.

In 1909, Ehrenfest worked at the Polytechnic Institute for almost a year, teaching differential equations of mathematical physics for two semesters. Disputes about his way of work and the procedures at the university combined with his personal background and related prejudices present in Russian society at the time resulted in his dismissal from the Polytechnic Institute.

Over a period of several years, Ehrenfest tried to find a position at different universities including the University of Czernowitz (now Tsjernovtsi), Ukraine, in 1910, and three universities in Germany, the University of Leipzig, the University of Munich, and the University of Berlin. Although highly recommended and supported by many important physicists, his attempts remained unrewarded.

It should be noticed that after attending a meeting of Zionists in Vienna in 1910, Ehrenfest became interested in this movement and highly enthusiastic about it.

Ehrenfest left St. Petersburg on January 6, 1912, traveling to Berlin to meet Max Planck and discuss two of his important papers with him

1. Ehrenfest, P., "Zur Frage nach der Entbehrlichkeit des Lichtäthers", *Phys. Zeit.*, Vol. 13, 1912, 317-319.

2. Ehrenfest, P., "Welche Zge der Lichtquantenhypothese spielen in der Theorie der Wrmestrahlung eine wesentliche Rolle?", *Annalen der Physik* Vol. 36, 1911, 91-118.

During the same trip, Ehrenfest visited several famous physicist while traveling to Munich, Zurich, and Prague, where he met Einstein.

When he returned to St. Petersburg in early March 1912, Ehrenfest found an opportunity to get a position as the Chair of Theoretical Physics at the University of Leiden, as the successor of Hendrik Antoon Lorentz. Ehrenfest worked there until his tragic end. For more details on the biography of Paul Ehrenfest, we refer the reader to

Einstein, A., "Paul Ehrenfest in memoriam", in *Out of My Later Years*, Secaucus, N.J., 1977.

Huijnen, P., Kox, A.J., "Paul Ehrenfests Rough Road to Leiden: A Physicists Search for a Position, 1904-1912", *Phys. Perspect*, 9, 2007, 186-211.

Hollestelle, M.J., "Paul Ehrenfest as a mediator", in M. Kokowski, (Ed.), *The Global and the Local: The History of Science and the Cultural Integration of Europe, Proceedings of the 2nd ICESHS*, Cracow, Poland, September 6-9, 2006, 787-792.

Figure 17: Paul Ehrenfest while teaching (photo taken from Mac Tutor archive).

4 A Translation of the Review by Paul Ehrenfest

L. Couturat, *The Algebra of Logic*, Translation from French with additions by Prof. I. Slišinski, Mathesis, 1909, pages 104+XIII, (Price 90 kopejka (kopek))

When presenting formal logic, it is necessary to know the following circumstances. The extraordinary tight classification of different types of reasoning and syllogisms, that is already developed in *knowledge*, founds in language a very difficult and inaccurate instrument to express it.

For this reason, in the theory of reasoning and syllogisms, it was accepted long ago that this classification should be expressed by conditional symbols.

This primarily concerns the subclassification of syllogisms into type "*A*, *E*, *I*, *O*" and the derivatives by reasonable symbols for 19 forms of regular syllogisms (from "Barbara" to "Ferison" in the 13th century). Later, a symbolic to represent different notions by circles in a plane was developed. The different ways of the mutual placement of circles correspond to different cases of combining two notions (premises) into a single conclusion.

The first symbolic notation, hardly better than stenography, unifies in a common picture all of the members of the syllogism, however, it is inflexible. The second symbolic notation is already considerably better: over a system of such circles, it is possible, after defining the corresponding rules, to perform transformations that also have a defined interpretation in logic.

(This can be compared with the fact that in chemistry formulas permit not only a systematic registration of different substances, but besides that, the transformations of formulas by predefined rules correspond to chemical reactions.)

It is easy to understand that this area will sooner or later lead a *speculative*-mathematical mind to the following question: Can these principles of symbolic notation, that appeared so fruitful in *operations over numbers and quantities* - "symbolic notation" - be transferred into *operations over all concepts*?

Now, regarding the attempts by Leibniz [1] and Grassman [2].

A wider development of the "*Algebra of Logic*" is due to two mathematicians: the Englishman George Boole (1815-1864) and the German E.

[1] Gottfried Wilhelm Leibniz, a German mathematician and philosopher.
[2] Hermann Günther Grassmann, a German linguist and mathematician.

Schröder (1841-1902), to whose further development mathematicians from all over the word contributed. In particular, the Russian mathematician P. Poreckij greatly assisted in the simplification of methods by means of his original formulation of the problem.

The "*Algebra of Logic*" first of all establishes a symbolic notation for these elementary actions which appear to be essential in operating with notions, similar to addition, equivalence, etc., which are used in operating with numbers and quantities. Further, the axioms upon which the entire formal logic is based, converts into the form of rules how to perform computations over these symbols, i.e., how a multitude of such symbols can be transformed into another multitude equivalent to it.

From understanding, if it is possible to say in this way, of the typographic character of these operations, we select exactly those symbols that were already long ago - with *a completely different meaning* - introduced into printing by mathematicians. This compliance - without which the review of the book on the algebra of logic on the pages of this journal would be impossible - often gives to the formulas in this discipline a paradoxical form at first sight, e.g.,

$$1 + 1 = 1, \quad (1)$$
$$A + AB = A, \quad (2)$$
$$AAA = A, \quad (3)$$
$$(A + \Gamma)(\Gamma + B)(B + A) = A\Gamma + \Gamma B + BA, \quad (4)$$

where the identities (2), (3), and (4) hold for an arbitrary choice of notions A, Γ, and B. For example, $A =$ all that is black, $\Gamma =$ all that is colored, $B =$ all that is firm.

The correctness of these equalities becomes understandable at the very same moment when the meaning of the operations used in the algebra of logic behind these symbols is explained.

($A\Gamma$) denotes "*all who belong at the same time to the class A and the class* Γ *(black colour)*.

The same applies to (ΓB) and (BA).

But, it is slightly difficult to use languages in these - from the point of view of logical relations - primitive constructions. In most cases, languages prohibit some propositions (coloured blackness?!). In other cases, the proposition - under the influence of different, arbitrarily added agreement - gives a completely different meaning to the words (shine silk = silk's shine).

$(A+\Gamma)$ denotes *all who belong to class A and also all who belong to class* Γ.

The most correct and reasonable expression of this "addition" would be, please, the following: all that belong to either A or B, or both at the same time (for example, "physicians and scholars").

It is now easy to verify this on examples of equalities (2) and (3).

For example, all coloured + all black balls = all coloured.

With the help of "multiplication" and multiple application of equalities (2) and (3), it is easy to verify (4).

1 denotes: *the universe of all thinkable thoughts.*

After that, equality (1) is obvious.

0 denotes: *classes, that do not contain any thinkable thoughts.* A' denotes "not A", i.e., *all that are not A.*

It is easy to verify the following statements:

1. $AA' = 0$, $A + A' = 1$,
2. $(A\Gamma)' = A' + \Gamma'$

$(A < \Gamma)$ denotes: *all A smaller than* Γ, which, however, can be expressed as:

$$A = x\Gamma \quad \text{or} \quad A\Gamma' = 0.$$

By using such expressions it is possible to express all the syllogisms in *an entirely numerical manner.* For example, the form of the syllogism "Ferison"

No man (L) is clairvoyant (M).	$L = xM'$
Some people are scholars (H).	$yL = zH$
Some scholars are not clairvoyant.	$zH = xyM'$.

By using the algebra of logic, it is possible to treat all syllogisms without intermediate constructions. In general, it is possible to reach the goal faster: first in the form of equalities we establish the entire system of given parcels. This system of reasoning is transformed into a unique system i.e., into an equivalent system, paying attention that the equality $A + \Gamma + B + \cdots = 0$ is equivalent to $A = 0$, $\Gamma = 0, \ldots$.

Furthermore, by using predefined rules, we calculate a system of reasoning - in some sense complete (!) - that follows from this given central reasoning. All of the computations are very simple, since in the algebra of logic - unlike classical algebra - action spaces do not spread until infinity. (For instance, "exponentiation" does not exists here - see equality (3)).

The area of application of the algebra of logic is further considerably extended due to the following observation.

In equalities in the algebra of logic, symbols $A, \Gamma, \ldots$, can represent particular notions, but also entire equalities that connect notions $L, M, \ldots$.

In this interaction, the equality

$$(LM' = L) = (LM = 0)$$

expresses in a unexpectedly short form the following *theorem* that can easily be verified on particular examples:

"A statement that the set of all L that are at the same time not M, is identical to the whole set L, is equivalent to the statement that none M belongs to the set L."

In the same way, the equality

$$(A + \Gamma + B = 0) = (A = 0)(\Gamma = 0)(B = 0)$$

formulates the above theorem on joining several logic equalities into a single equality.

The goal of all these remarks is to give a description from another angle of a discipline which was introduced in the not very extensive book by Couturat.

It can be considered as an introduction to the algebra of logic in the sense that the author does not assume any background knowledge, except familiarity with general notions of logic, and at the same time presents to the reader all the questions that are foundations of a very extensive (the work of Schröder consists of four large volumes) literature on this subject.

Reading this book, on the other hand, requires very serious work, since the author does not restrict the presentation to the general presentation of the symbolic method, but *uses it* - already from page 7 - for the presentation and derivation of all the theorems. In this manner, this book cannot just be read, it is required to perform computations over entire pages, while reading. Besides that, the point of view presented in the book is at a high level of abstraction, the book is written for a French reader - a mathematician. (In this latter respect, additional remarks in the Russian edition make it considerably easier for the Russian reader.)

For an initial familiarization with the subject, it could be preferable to read several former publications [3]. The presentation of the symbolic method

[3] For example, E. Schröder, *Operationskreis des Logikcalculus*, Teubner, 1877.

in this book, on the other hand, offers the reader a possibility to be convinced that this method in any case has three advantages:

1. The book offers a possibility to distinctively denote the set of all propositions, upon which some conclusions are based, in such a way that the introduction of unconscious assumptions, that are often met in the formulation of reasoning, are almost completely excluded. In reality, axioms upon which formal logic is based appear in distinct and completely unusual forms.

2. Consequently, the formulation of any reasoning in terms of logic equalities is at least 5 to 10 times shorter than the literal formulation, which is an admirably concise presentation.

3. The symbolic formulation provides the possibility of "computing" conclusions from such complex systems of propositions, for which a literal presentation is almost or completely impossible.

Fortunately, we have already lost the habit of requiring that each mathematical speculation needs "practical usefulness". It is however not less appropriate to tackle the question whether in physics or technique such complex systems of propositions exists. *We think that we should answer these questions affirmatively.* Example: Let the task be to design networks of connections in automatic telephone stations. It is necessary to determine the following: 1) If the station will work correctly for an arbitrary combination of possible occurrences in the working station. 2) If the station contains some redundancies.

Each of these combinations is a proposition, each small commutator is a logic "Or-Or", all together - a system of qualitative (non quantitative) "propositions", leaving nothing more to be desired regarding complexity and intricacy.

Does it follow that, when solving such problems, every time some ingenious method - in many cases just a simple routine method - of *trials on a graph* should be used?

Is it right, that regardless of the existence of the already elaborated *algebra of logic*, the specific *algebra of switching networks* should be considered as a utopia?

P. Ehrenfest

5 A Reprint of the Review by Ehrenfest

This section contains the reprint of the original review of P. Ehrenfest as it appeared in

Ehrenfest, P., "Review of Couturat's Algebra logiki", *Žurnal Russkago Fiziko-hemiceskago Obščestva, Fizičeskij otdel*, Vol. 42, 1910, Otdel vtoroj, 382-387.

Библіографія.

Л. Кутюра. *Алгебра Логики.* Переводъ съ французскаго съ прибавленіемъ проф. *И. Слешинскаго.* Mathesis. 1909. 104 + XIII стр. (цѣна 90 к.).

При изложеніи формальной логики даетъ себя знать слѣдующее обстоятельство: чрезвычайно тонкая классификація раз-

[1]) Гальванитъ впервые былъ демонстрированъ у насъ А. Л. Гершуномъ въ засѣданіи Р. Ф. О. 12 октября 1910 г. Въ настоящее время его уже можно получать въ Петербургѣ—въ шведскомъ промышленномъ магазинѣ, Литейный, 24.

This page presents page 382 of the review, the text on pages 383 to 387 is continued on the following pages. [4]

[4]For the work of the Russian mathematician P.S. Poreckij mentioned by Ehrenfest, see, for instance Stanković, R.S., Astola, J.T., (eds.), *Reprints from the Early Days of Information Sciences, On the Contributions of P.S. Poreckij to Switching Theory*, TICSP # 46, 2009, ISBN 978-952-15-1980-2 (remark by the editors).

личныхъ типовъ сужденій и умозаключеній, выработанная уже въ *сознаніи*, находитъ въ *языкѣ* слишкомъ тяжеловѣсный и неточный инструментъ для своего выраженія.

Поэтому въ ученіи о сужденіяхъ и умозаключеніяхъ уже давно принято передавать эту классификацію условными символами.

Сюда относятся прежде всего подраздѣленіе сужденій на типы „*А, Е, И, О*" и произведенные изъ нихъ словесные символы для 19 формъ правильныхъ умозаключеній (отъ „Барбара" до „Феризонъ" съ 13-го столѣтія). Позднѣе развилась символика, представляющая различныя понятія при помощи круговъ на плоскости: различные способы относительнаго расположенія круговъ соотвѣтствуютъ различнымъ случаямъ соединенія двухъ понятій въ одно сужденіе.

Первая символика едва ли болѣе, чѣмъ стенографія, объединяющая въ одну картину всѣ члены сужденія, но негибкая. Вторая символика даетъ уже гораздо большее: надъ системой такихъ круговъ можно по опредѣленнымъ правиламъ производить преобразованія, имѣющія также опредѣленную интерпретацію въ логикѣ. (Сравнить съ этимъ тотъ фактъ, что формулы химіи доставляютъ не только систематическую регистратуру различныхъ веществъ, но что кромѣ этого—преобразованія этихъ формулъ, произведенныя по опредѣленнымъ правиламъ, соотвѣтствуютъ химическимъ преобразованіямъ).

Само собою разумѣется, что эта область рано или поздно должна была побудить *спекулятивно*-математическіе умы къ слѣдующей попыткѣ: тотъ принципъ символическихъ обозначеній, который оказался столь плодотворнымъ для *дѣйствій надъ числами и величинами*—„буквенное исчисленіе" — перенести на *дѣйствія надъ всякими понятіями*.

Сюда относятся уже попытки Лейбница и Грассмана. Болѣе широкимъ развитіемъ обязана „*алгебра логики*" двумъ математикамъ: англичанину Джорджу Булю (1815—1864) и нѣмцу Е. Шредеру (1841—1902); въ дальнѣйшей разработкѣ принимали участіе математики всѣхъ странъ. Въ частности русскій математикъ П. Порѣцкій много содѣйствовалъ упрощенію методовъ своей оригинальной постановкой вопроса.

„*Алгебра логики*" прежде всего устанавливаетъ символическія обозначенія для тѣхъ элементарныхъ дѣйствій, которыя являются такими же основными при оперированіи съ понятіями, какъ сложеніе, приравниваніе и т. п. при оперированіи съ числами и величинами. Далѣе, аксіомы, на которыхъ основана вся формальная логика, облекаются въ форму *правилъ о томъ, какъ надъ этими символами производить вычисленія*, т. е. какъ одинъ комплексъ такихъ символовъ преобразовать въ другой, эквивалентный ему.

Изъ соображеній, если можно такъ выразиться, типографскаго характера для этихъ операцій выбраны тѣ самые значки, которые уже давно—*съ совершенно инымъ значеніемъ*—были математиками введены въ печать. Это соглашеніе—безъ котораго на страницахъ настоящаго журнала былъ бы невозможенъ рефератъ по алгебрѣ логики—придаетъ формуламъ этой дисциплины нерѣдко на первый взглядъ парадоксальный видъ, напр.:

$$(1)\ 1+1=1; \qquad (2)\ A+AБ=A$$

$$(3)\ AAA=A;\ (4)\ (A+Б)\ (Б+В)\ (В+A)=AБ+БВ+ВA,$$

причемъ равенства (2), (3) и (4) имѣютъ мѣсто при произвольномъ выборѣ «понятій A, $Б$ и $В$. Напр., $A=$ «все, что черно», $Б=$ «все, что шарообразно», $В=$ «все, что твердо».

Но справедливость этихъ равенствъ тотчасъ станетъ понятна, какъ только будетъ объяснено, какія операціи подразумѣваются въ алгебрѣ логики подъ значками, обозначающими дѣйствія.

($AБ$) обозначаетъ: *„все, что одновременно принадлежитъ классу A и классу $Б$* (черный шаръ).

Тоже относительно ($БВ$) и ($ВA$).

Впрочемъ, языкъ слишкомъ тяжеловѣсенъ, чтобы поспѣть за этой—въ логическомъ отношеніи—примитивной конструкціей; въ большинствѣ случаевъ онъ запрещаетъ перестановку (шарообразная чернота?!); въ другихъ случаяхъ перестановка—подъ вліяніемъ различныхъ, случайно сложившихся соглашеній, придастъ словамъ совсѣмъ другой смыслъ (блестящій шелкъ = шелковый блескъ).

($A+Б$) обозначаетъ: *все, что принадлежитъ классу A, и сверхъ того все, что принадлежитъ классу $Б$.*

Наиболѣе точное словесное выраженіе этого «сложенія» будетъ, пожалуй, слѣдующее: все, что принадлежитъ или къ A, или къ $Б$, или къ обоимъ вмѣстѣ (примѣръ: «врачи и ученые»).

Теперь нетрудно на примѣрахъ провѣрить равенства (2) и (3).

Напр., все шарообразное + всѣ черные шары = все шарообразное.

При помощи „умноженія“ и многократнаго примѣненія равенствъ (2) и (3) легко провѣрить (4).

1 обозначаетъ: *совокупность всего мыслимаго.*

Послѣ этото равенство (1) очевидно.

O обозначаетъ: *классъ, не содержащій ничего мыслимаго.*

A' обозначаетъ «Не—A», т. е. *все, что не есть* A.

Легко провѣрить слѣдующія утвержденія:

$$AA' = 0; \ A + A' = 1.$$

$$(AБ)' = A' + Б'.$$

$(A < Б)$ обозначаетъ: *всѣ* A *суть* $Б$, что, впрочемъ, можно выразить и такъ:

$$A = xБ \text{ или } AБ' = 0.$$

Съ помощью такихъ пріемовъ можно уже *чисто вычислительнымъ путемъ* вывести всѣ силлогизмы, напр., форма «Феризонъ»

Ни одинъ человѣкъ (Л) *не всевѣдущъ* (M) . . . . . $Л = xM'$

Нѣкоторые люди – ученые (H) . . . . $yЛ = zH$

Нѣкоторые ученые не всевѣдущи . . . . $zH = xyM'$

Но, пользуясь алгеброй логики, можно обойтись безъ послѣдовательнаго построенія всѣхъ силлогизмовъ. Обыкновенно можно гораздо короче придти къ цѣли: сперва въ формѣ равенствъ устанавливается вся система данныхъ посылокъ. Эта система сужденій преобразуется въ одно единственное, имъ эквивалентное; при этомъ принимается во вниманіе, что утвержденіе $A+Б+В+\ldots=0$ равносильно съ $A=0$, $Б=0, \ldots$.

Далѣе по опредѣленнымъ правиламъ «вычисляется»—въ извѣстномъ смыслѣ—полная (!) система сужденій, которыя слѣдуютъ изъ этого центральнаго сужденія. Всѣ вычисленія очень просты, такъ какъ въ алгебрѣ логики—въ противоположность обыкновенной алгебрѣ—кругъ дѣйствій не разростается до безконечности. (Напр., здѣсь не существуетъ «степеней»—см. равенство (3)).

Область примѣненія алгебры логики значительно расширяется еще въ виду слѣдующаго соображенія.

Въ равенствахъ алгебры логики символы A, $Б, \ldots$ могутъ представлять не только отдѣльныя понятія, но и *цѣлыя равенства, связывающія понятія* $Л$, $M, \ldots$

При такой интерпретаціи равенство

$$(ЛM' = Л) = (ЛM = 0)$$

выражаетъ въ неожиданно короткой формѣ слѣдующую *теорему*, которую легко провѣрять на частныхъ примѣрахъ:

3

«Утвержденіе, что совокупность всѣхъ *Л*, которыя въ то же время не—*М*, совпадаетъ съ совокупностью всѣхъ вообще *Л*, равносильно съ утвержденіемъ, что ни одно *М* не принадлежитъ къ классу *Л*».

Точно также равенство:

$$(A + Б + В = 0) = (A = 0)\ (Б = 0)\ (В = 0)$$

формулируетъ вышеупомянутую теорему о соединяемости нѣсколькихъ логическихъ равенствъ въ одно.

Всѣ эти указанія имѣютъ цѣлью хотя бы съ внѣшней стороны охарактеризовать технику своеобразной дисциплины, введеніемъ въ которую является небольшая кнчга Кутюра.

Введеніемъ въ алгебру логики она можетъ считаться въ томъ смыслѣ, что авторъ не предполагаетъ у читателя никакихъ предварительныхъ свѣдѣній, за исключеніемъ знакомства съ общими понятіями логики, и въ то же время показываетъ читателю всѣ вопросы, которые легли въ основу весьма обширной (сочиненіе Шредера обнимаетъ 4 большихъ тома) литературы этого предмета

Но съ другой стороны чтеніе этой книги требуетъ *весьма* серьезной работы, такъ какъ авторъ не ограничивается общимъ *изложеніемъ* символическаго метода, а *пользуется* имъ—начиная ъ 7-ой страницы—для изложенія и вывода всѣхъ теоремъ. Такимъ образомъ эту книгу нельзя просто прочитывать, а необходимо на протяженіи цѣлыхъ страницъ сопровождать чтеніе ея производствомъ вычисленій. Кромѣ того, точка зрѣнія ея чрезвычайно абстрактна—она написана для французскаго читателя—математика. (Въ послѣднемъ отношеніи примѣчанія русскаго изданія представляютъ *весьма цѣнное* облегченіе для читателя).

Поэтому для бѣглаго ознакомленія съ предметомъ придется, можетъ быть, предпочесть нѣкоторыя прежнія сочиненія [1]). Но съ другой стороны какъ разъ это изложеніе по символическому методу даетъ читателю возможность убѣдиться, что онъ во всякомъ случаѣ имѣетъ три преимущества:

1. Онъ даетъ возможность отчетливо обозначить всю совокупность предположеній, на которыхъ основываются какіе либо выводы, такъ что введеніе безсознательныхъ допущеній, такъ часто встрѣчающееся при словесной формулировкѣ разсужденія, *почти* совершенно исключается. Въ частности, аксіомы, на которыхъ

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[1]) Напр., E. Schröder, Operationskreis des Logikcalculs. Teubner. 1877.

основывается формальная логика, выступаютъ въ отчетливой и притомъ совершенно непривычной формѣ.

2. Вслѣдствіе того, что формулировка всякаго разсужденія при помощи *логическихъ равенствъ* по крайней мѣрѣ въ 5—10 разъ короче словесной, достигается удивительная сжатость изложенія.

3. Символическая формулировка даетъ возможность «вычислять» слѣдствія изъ такихъ сложныхъ системъ посылокъ, въ которыхъ при словесномъ изложеніи почти или совершенно невозможно разобраться.

Къ счастью, уже отвыкли требовать отъ каждой математической спекуляціи прежде всего «практической пользы». Тѣмъ не менѣе, быть можетъ, умѣстно коснуться вопроса о томъ, не встрѣчаются ли въ физикѣ или въ техникѣ въ самомъ дѣлѣ такія сложныя системы посылокъ. *Мнѣ думается, что на этотъ вопросъ слѣдуетъ отвѣтить утвердительно.* Примѣръ: пусть имѣется проектъ схемы проводовъ автоматической телефонной станціи. Нужно опредѣлить: 1) будетъ ли она правильно функціонировать при любой комбинаціи, могущей встрѣтиться въ ходѣ дѣятельности станціи; 2) не содержитъ ли она излишнихъ усложненій.

Каждая такая комбинація является посылкой, каждый маленькій коммутаторъ есть логическое «или—или», воплощенное въ эбонитѣ и латуни; все вмѣстѣ—система чисто качественныхъ (въ сѣти слабаго тока именно *не* количественныхъ) «посылокъ», ничего не оставляющая желать въ отношеніи сложности и запутанности.

Слѣдуетъ ли при рѣшеніи этихъ вопросовъ разъ навсегда удовлетвориться геніальнымъ—а по большей части просто рутиннымъ—способомъ *пробованія на графикѣ*?

Правда ли, что несмотря на существованіе уже разработанной «алгебры логики» своего рода «алгебра распредѣлительныхъ схемъ» должна считаться утопіей?

П. Эренфестъ.

А. В. Цингеръ. *Начальная физика.* Первая ступень. XX+499 стр. Москва.

„Первая ступень“ курса физики, разсчитаннаго авторомъ на двѣ части, представляетъ собою попытку „дать элементарный очеркъ физики, имѣющій цѣлью выяснить начинающимъ немногіе

4

6 Reviews about Ehrenfest

In this section, we reprint reviews about the work of scholars where the remarks of P. Ehrenfest on applicability of algebra of logic in the design of logic networks is mentioned. These are reviews of

1. Alonso Church about the article by of T.A. Kalin,
2. Alonso Church about papers by A. Nakashima and M. Hanzawa,
3. D.D. Comey and G.L. Kline about the article of A.A. Zinoviev,
4. G.L. Kline about the article of S.N. Anovskaa.

6.1 Review by Alonso Church for T.A. Kalin mentioning Ehrenfest

Church, A., "Review of *Formal Logic and Switching Circuits* by Theodore A. Kalin", *The Journal of Symbolic Logic*, Vol. 18, No. 4, December 1953, 345-346.

THEODORE A. KALIN. *Formal logic and switching circuits.* ***Proceedings of the Association for Computing Machinery, Jointly sponsored by the Association for Computing Machinery and the Mellon Institute, Pittsburgh, Pa., May 2 and 3, 1952,*** photo-offset, Richard Rimbach Associates, Pittsburgh 1952, pp. 251–257.

"The historical development of the application of symbolic logic to calculating machinery is traced to the present, and a brief sketch of some modern developments is presented." The author recounts briefly, in descriptive fashion, the invention by Charles Babbage of a scheme of mechanical notation by which the various parts of a mechanism are expressed on paper by symbols, and the development of the algebra of logic by Boole and Jevons (only these two names are mentioned in the latter connection). Shannon's IV 103(3) (reprinted XVIII 347(1)) is referred to for the application of Boolean algebra or propositional calculus — the distinction between them is not important in this connection — to relay and switching circuits, and Hartree's ***Calculating instruments and machines*** (XVIII 347) is mentioned as using a like method (in connection with computing and control circuits). The final half of the paper under review is then devoted to an account in more detail of the content of ***Synthesis of electronic computing and control circuits*** (XVIII 347), which uses, not Boolean algebra in the modern sense, but a numerical representation of it which is in fact the same as the original algebra of Boole (19***1, 2, 3***). The author concludes: "There can be little doubt that formal symbolic design methods will grow in utility and neatness of application as new circuital techniques are adopted and as new switching elements are discovered and put to work. It is almost true by definition that digital systems operating by means of 'on-off' devices will continue to offer a most enticing challenge to two-valued logic, and it is our task to so refine our exact methods that the designer shall be increasingly free to devote his skill and experience to areas less subject to routine effort."

The author's statement that Shannon's paper of 1938 is the first exposition of the relations between "two-valued logic and switching circuitry" requires some qualification. It seems to be not generally known that the first suggestion of such a relationship was made in 1910 by Erénfést, in his review (186½1) of Couturat's 10020A. According to Ánovskaá (XVI 46), details of Erénfést's proposal were worked out by Šéstakov in 1934–35 but not published until 1941. Meanwhile the same idea had been reached independently by Nakasima and Hanzawa in 1936 (see the review next following).

The reviewer has not seen Erénfést's review, and is indebted to George L. Kline for information as to its content. He is also indebted to George W. Patterson for calling his attention to the papers of Nakasima and Hanzawa, and others published in Japan.

ALONZO CHURCH

AKIRA NAKASIMA and MASAO HANZAWA. *The theory of equivalent transformation of simple partial paths in the relay circuit.* **Nippon electrical communication engineering** (Tokyo), no. 9 (February 1938), pp. 32–39.

AKIRA NAKASIMA. *The theory of four-terminal passive networks in relay circuit.* Ibid., no. 10 (April 1938), pp. 178–179.

AKIRA NAKASIMA. *Algebraic expressions relative to simple partial paths in the relay circuit.* Ibid., no. 12 (September 1938), pp. 310–314.

AKIRA NAKASIMA. *The theory of two-point impedance of passive networks in the relay circuit.* Ibid., no. 13 (November 1938), pp. 405–412.

AKIRA NAKASIMA. *The transfer impedance of four-terminal passive networks in the relay circuit.* Ibid., no. 14 (December 1938), pp. 459–466.

AKIRA NAKASIMA and MASAO HANZAWA. *Expansion theorem and design of two-terminal relay networks (Part I).* Ibid., no. 24 (April 1941), pp. 203–210.

Nippon electrical communication engineering publishes condensed English translations, and abstracts in English, of papers which were previously published in Japanese in the **Journal of the Institute of Electrical Communication Engineers of Japan.** The first of the above papers, for example, is described as a condensed translation of a paper which appeared in two parts in the latter periodical, no. 165 (December 1936) and no. 167 (February 1937). The reviewer has not seen the Japanese originals of the papers.

The six papers are concerned with developing and applying an algebra of partial paths in relay circuits, which is in fact identical with the "symbolic relay analysis" that was later introduced by Shannon, and dual to the "algebra of switching circuits" of Erénfést and Šéstakov (see the preceding review).

The first paper introduces the algebra by providing that if A and B are simple partial paths (two-terminal circuits), then $A + B$ shall represent the series connection of A and B, and AB the parallel connection of A and B; $A = B$ shall mean that the acting functions of A and B are equal, i.e., that A is open when B is open and closed when B is closed; $\bar{A}$ shall be a simple partial path which is open when A is closed and closed when A is open; p and s shall be simple partial paths which are always open and always closed respectively (or, as the authors say, give always infinite impedance and zero impedance respectively). Many laws of the algebra are developed which in fact coincide with familiar laws of Boolean algebra, but the authors do not state that the algebra is a Boolean algebra.

In the third paper (of which the Japanese version was published in August 1937) the algebra is reduced to an algebra of sets by making correspond to each simple partial path the set of (in effect) times at which its impedance is infinite, so that "theorems and expressions developed in the theory of set may, therefore, be applied to acting impedance problems of simple partial paths." In the sixth paper the authors make explicit reference for the first time to Boole (193) and Schröder (427); the expansion theorem mentioned in the title of this paper is Boole's law of development (193, pp. 72–75), as the authors point out.

ALONZO CHURCH

6.2 Review by Alonso Church for A. Nakashima and M. Hanzawa

Church, A., "Review of *Formal Logic and Switching Circuits* by Theodore A. Kalin", *The Journal of Symbolic Logic*, Vol. 18, No. 4, December 1953, 346.

AKIRA NAKASIMA and MASAO HANZAWA. *The theory of equivalent transformation of simple partial paths in the relay circuit.* ***Nippon electrical communication engineering*** (Tokyo), no. 9 (February 1938), pp. 32–39.

AKIRA NAKASIMA. *The theory of four-terminal passive networks in relay circuit.* Ibid., no. 10 (April 1938), pp. 178–179.

AKIRA NAKASIMA. *Algebraic expressions relative to simple partial paths in the relay circuit.* Ibid., no. 12 (September 1938), pp. 310–314.

AKIRA NAKASIMA. *The theory of two-point impedance of passive networks in the relay circuit.* Ibid., no. 13 (November 1938), pp. 405–412.

AKIRA NAKASIMA. *The transfer impedance of four-terminal passive networks in the relay circuit.* Ibid., no. 14 (December 1938), pp. 459–466.

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ALONZO CHURCH

6.3 Review by D.D. Comey and G.L. Kline for A.A. Zinoviev mentioning Ehrenfest

Comey, D.D., Kline, G.L., "Review of *Rasirat tematiku logiceskih issledovanij (Broaden the subject matter of logical investigations). by A.A. Zinov'ev", (Rabote seminara po logike v Institute Filosofii AN SSSR(On the Work of the Seminar on Logic in the Institute of Philosophy of the Academy of Sciences of the USSR by A.A. Zinov'ev))*, *The Journal of Symbolic Logic*, Vol. 24, No. 3, September 1959, 232-233.

A. A. ZINOV'ÉV. *Rasširat' tématiku logičéskih isslédovanij* (Broaden the subject matter of logical investigations). Ibid., no. 3 (1957), pp. 211–215.

A. A. ZINOV'ÉV. *O raboté séminara po logiké v Instituté Filosofii AN SSSR* (On the work of the seminar on logic in the Institute of Philosophy of the Academy of Sciences of the USSR). Ibid., no. 2 (1958), pp. 167–172.

Zinov'év's two papers, although published nearly a year apart, constitute a single report on the logic seminar of the Institute of Philosophy in Moscow. This seminar was organized in September 1956 (i.e., at the high point of the Soviet cultural "thaw" of that period). Zinov'év explains that such a seminar was needed because, "until very recently there has been little attention to the development of logic in philosophical institutions; the range of interests of professional logicians has been narrowed to the limits of the content of obsolete textbooks" [i.e., Soviet texts in purely Aristotelian logic] (p. 211).

Since a chief purpose of the seminar was to bring Soviet philosophers up to date, many of the reports are historical in character. Ruzavin and Stážkin survey the development of mathematical logic from its beginnings in Stoic and medieval logic. Finn and Lahuti, in a joint paper, outline recent British and American work in the theory of induction, modal logic, the logic of explanation, etc. Later reports are of greater theoretical interest: Povarov discusses combinatory logics, distinguishing "technical logic" from the "algebra of logic," and pointing out that "the possibility of technical applications of mathematical logic" (i.e., the application of Boolean algebra to the analysis of electric relay-contact circuits) was first noted by the Russian physicist P. S. Erénfést in 1910 (cf. the review XVI 46). Stážkin, in a second paper, explores the status of logical and semantic paradoxes. He leaves open the question as to whether they can be resolved by the elimination of "semantic terms" (p. 168). But he explicitly denies that *all* semantic paradoxes can be resolved merely by distinguishing between "use" and "mention" of expressions, or by modifying the rules for logical negation.

Lahuti relates Ackermann's "strenge Implikation" (cf. XXII 327) to Russell's "material implication" and Lewis's "strict implication," asserting that Ackermann's, and Ánovskaá's, methods make it possible to eliminate most, though not all, of the paradoxes in SI. Ackermann's system has the further merit of accommodating supplementary non-logical axioms and of being extendable to the predicate calculus. — Povarov discusses group invariance, including symmetry, of Boolean functions. — Finn reports on information theory and "machine logic," distinguishing a *general* information language from *special* information languages — those of geometry, chemistry, etc. He symbolizes the "logical processing of information" as follows: $Q_1, \ldots, Q_n \vdash i$; $Q_1 \neq i$, $Q_i \neq i$, $Q_{i+1} \neq i$, where "Q" is the "coded" message and "i" is the desired information. Finn also distinguishes several kinds of "functors," e.g., "operators" (which form names from names), "predicators" (which form propositions from names), "connectors" (which form propositions from propositions).

Zinov'év's own paper on "paradoxes of indefiniteness" is reproduced in fullest detail. It sets forth a system to avoid the paradox of the set of all normal sets and other related antinomies of set theory. However, Zinov'év's system uses such a vague criterion of existence for sets (given in terms of an equally obscure concept of the existence of a proposition), that the paper itself suffers from indefiniteness. For instance, the definition of a set on page 171 presupposes, in the definiens, the concept represented by the definiendum. Again, on page 170 it is stated that if Π does not exist, then every statement containing [Π] is indefinite; but a few paragraphs later we are told that if Π does not exist, then both the statements [Π does not exist] and [Π exists] are definite, and are the only two definite statements containing [Π]. Yet it is plain that any number of disjunctive propositions containing [Π does not exist] as one member would be definite by Zinov'év's original criterion.

Zinov'év often fails to distinguish between use and mention of an expression, and, although he proscribes "non-effective" proofs by *reductio ad absurdum*, he appeals to the principle that either X^1 does exist or it does not. The proof given on pages 171–172 is invalid: The author constructs a sequence of sets $M^1, M^2, \ldots$ such that for each i there is a property X^i which holds for $M^1, M^2, \ldots, M_1, \ldots, M_{i-1}$, but does not hold for M_i. From this he erroneously infers that there cannot be any property X which holds for all the sets $M^1, M^2, \ldots, M_1, M_2, \ldots$. In fact, if M^1 is taken as the null set and M^2 as the set containing only M^1 as a member, then both M^1 and M^2 are normal. Using only these two sets, we can go on to form $M_1, M_2, \ldots$ according to his method. If we then take our construction as the construction of the integers in set theory, M^1 is zero, M^2 is unity, and $M_1, M_2, \ldots$ are two, three, ... Quite obviously there are properties which hold for all of $M^1, M^2, M_1, M_2, \ldots$; e.g., they are all integers; they are all less than two or else uniquely factorable into prime factors.

These Soviet discussions reveal a detailed familiarity with current work in mathematical logic and information theory outside the Soviet Union. They are also marked by a striking preference for non-Russian technical terms (e.g., *antécédént, déskriptivnyj, distinkciá, informaciá, intérprétaciá, kod* [code]) in cases where there are perfectly good Russian equivalents. DAVID D. COMEY and GEORGE L. KLINE

6.4 Review of G.L. Kline for S.N. Anovskaja

Kline, G.L., "Review of *Foundations of Mathematics and Mathematical Logic* by S. A. Anovskaa", *The Journal of Symbolic Logic*, Vol. 16, No. 1, March 1951, 46-48.

S. A. Ánovskaá. ***Osnovaniá matématiki i matématičéskaá logika*** **(Foundations of mathematics and mathematical logic).** ***Matématika v SSSR za tridcat' lét 1917–1947*** **(Mathematics in the USSR for the thirty years 1917–1947), OGIZ, Moscow and Leningrad 1948, pp. 9–50.**

The present paper is a survey of the work in mathematical logic and the foundations of mathematics done in the Soviet Union between 1917 and 1947, with some reference to pre-revolutionary mathematicians and logicians. Soviet mathematicians, the author emphasizes, reject the view of Poincaré, Heyting, et al., that "the propositions of pure mathematics say nothing about reality." On the Soviet view, the formal axiomatic systems of mathematics—which admit of many qualitatively different interpretations—rest on a material [*sodéržatél'naá* = *inhaltliche*] arithmetic, in which numbers and the relations between them are univocally defined. The spatial forms and quantitative relations of the material world are the specific subject-matter of mathematics (Engels). In contrast to Carnap and the logical positivists, A. N. Kolmogorov asserts that a formal apparatus is valid only when it "corresponds to a real content."

Turning to the historical development of mathematical logic, the author mentions the work of De Morgan, Boole, Jevons, Peirce, and Schröder, and goes on to state that "the culmination of this period . . . was the work of the Russian logician, P. S. Poréckij, Lobačévskij's colleague at Kazan University" (p. 19). Poréckij considered his 582 the first attempt at a complete theory of qualitative inference (by a "quality" Poréckij meant what is now called a "one-place predicate").

Soviet mathematicians, according to the author, deny that there is any "crisis" in the foundations of mathematics, although they recognize real difficulties in connection with the applicability of the laws of formal logic, extrapolated from finite domains, to infinite domains, especially the law of excluded middle; and the paradoxes of mathematical logic and set theory. The chief interest of Soviet writers, we are told, is in the application of mathematical logic to special problems in mathematics and technology.

One of the first treatments of the law of excluded middle, according to Ánovskaá, is to be found in the introduction to S. O. Šatunovskij's algebra textbook, published in Odessa in 1917. From the excerpts which she reproduces, it is evident that Šatunovskij did have ideas on the subject similar to those of Brouwer, although they seem to be less clearly formulated. However, it should be noted that Brouwer's publications along this line go back to 1908.

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The problem of the excluded middle was discussed in 1925 by Kolmogorov in his 314*1*, which, the author maintains, anticipated Gödel's result of 418*11* (1932) to the effect that "die intuitionistische Arithmetik und Zahlentheorie nur scheinbar enger ist als die klassische." Kolmogorov offers an interpretation of classical arithmetic for which—through replacement of every variable by its double negation—all of its "known" propositions become propositions of intuitionistic arithmetic. He further states: "The application of the principle of excluded middle will never lead to a contradiction. In fact, if a false formula were obtained with its help, the corresponding formula of pseudo-mathematics would be demonstrated without its help, and would nevertheless lead to a contradiction." (314*1*, p. 661.)

In 1929 V. I. Glivénko in his 381*2* obtained a stronger result than Kolmogorov's regarding the propositional calculus. He showed that if a formula is demonstrable in the classical propositional calculus its double negation is demonstrable in the intuitionistic calculus, and that if the negation of such a formula is classically demonstrable it is also intuitionistically demonstrable.

Ánovskaá emphasizes that the intuitionistic logic, formulated as a finite set of axioms and rules of inference, can be "adequately applied to a given domain of objects" quite independently of the epistemological premises of intuitionism. Soviet writers accept the calculus and build upon it, but they reject the "idealistic" epistemology. The real significance of Heyting's logic—as a calculus of problem-solving—was exhibited by Kolmogorov in his 314*2*.

The work 304*1* of Šatunovskij's pupil, M. I. Schönfinkel, written in 1920 and published in 1924, is justly stressed as a major advance in the development of mathematical logic. However, Ánovskaá also credits Schönfinkel with originating the idea of a function as a special abstract object distinct from its values; in fact, this notion goes back at least to Frege (though admittedly with a difference in terminology which is somewhat misleading). The author states that "Schönfinkel's ideas were widely taken up by American mathematicians, first by Curry, who constructed his 'combinatory logic' (1930) on their foundation, and by Church, whose calculus of λ-conversion represents a certain 'formalization' of Schönfinkel's ideas" (p. 33). Without minimizing the significance of Schönfinkel's radically new idea, it should be noted that Curry has carried its development to a point far beyond the bare beginning made by Schönfinkel, and has contributed important additional ideas without which this development would have been impossible; also that Church's calculus of λ-conversion is not a mere variation of the Schönfinkel-Curry calculus of combinators, but represents a different approach. Its relationship to the work of Schönfinkel and Curry is made clear in Church's VI 171(1) (cf. pp. 3–5, 43–51).

Schönfinkel's 367*1*, written in collaboration with Bernays, is fairly represented as one of the important early papers on the decision problem. (The author gives the biographical information that Schönfinkel became mentally ill and died in Moscow in 1942.)

Ánovskaá further summarizes works of Žégalkin 344*1*, V 69 (1) (who died on March 28, 1947); Novikov XI 129 (3), XIII 170 (1), XIV 255 (4); Bočvar IV 98 (2), V 119 (1), XI 129 (1, 2), XII 27 (1); Malcév II 84 (2); and Markov XIII 52 (2), XIII 53 (1).

The idea of applying Boolean algebra to the analysis of electrical relay-contact circuits, the author points out, was first put forward in 1910 by the Russian physicist Erénfést in a review 186½*1* of the Russian translation (100*20*A) of Couturat's ***L'algèbre de la logique***. Erénfést's proposal for an "algebra of switching circuits" was worked out in detail in 1934–35 by Glivénko's pupil, V. I. Šéstakov, whose results were embodied in a paper written in January 1935. This paper, according to Ánovskaá, was not published at the time, but formed the basis of Šéstakov's candidate's dissertation, the major part of which was published in 1941 in the journal ***Téhničéskaá fizika*** (11:6). In the meantime (1938), Shannon had published his paper IV 103 (3) and gained credit for the idea.

This problem was further explored by A. M. Gavrilov in papers published between 1945 and 1947. Šéstakov's later work XII 135 (1), according to Ánovskaá, is evidence that even many-valued logics have practical significance and technological application.

In the concluding pages the author summarizes papers by Markov XIV 67 (1) on recur-

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