Technical Background on the Fundamental Noise Problem

The backdrop to the development of the two-sample (Allan) Variance was the fundamental time domain problem that was the trigger for my Master's thesis back in 1965. When you go to school and take statistics, you learn about the classical variance and standard deviation. But when you encounter clock noise it turns out that the classical variance does not converge for the kinds of noise we see. For example, it does not converge for flicker-noise frequency modulation (FM), and you see this kind of noise all the time in nature. Being able to deal with flicker noise is a very important problem not only in time and frequency, but in analyzing many other natural processes. It's very ubiquitous in nature.

You see flicker in semiconductor junctions, resistors, neurons, and music. As an unusual example, the river Nile at flood stage is modeled by a flicker noise process over the two thousand years for which data are available. We have a Big Spring that provides water for the community where we live as well as for irrigation. I measured its annual flow over fifty-two years and it's modeled by flicker noise. I attended an international conference in 1977 in Tokyo, where people came from all over the world talking about flicker noise and where it is observed. It was a really fun conference. I was asked to come and talk about flicker noise in atomic clocks. Professor Musha, who invited me from the University of Tokyo, shared how traffic flow is a flicker-noise process.

So, for flicker noise, calculating the classical variance as the integral of the spectrum, the integral blows up at zero and infinity. So, the classical variance grows without limit for flicker noise and also has no well-defined mean value.

When you do a classical variance analysis of the clock, you cannot distinguish the kinds of noise that are present. You end up with a value and an amplitude, and you don't really know what's going on with the clocks. This was a problem that we encountered back in 1965, which, Dr. Barnes, my great mentor and colleague sent me off to work on in conjunction with my thesis.

Because of the problem of characterizing these instabilities in quartz oscillators and atomic clocks, there was a special conference held at NASA Goddard, in Beltsville, Maryland, sponsored by the IEEE and by NASA, and inviting people to present papers and discuss this problem of how you characterize the short-term and long-term instabilities of these precision oscillators, with flicker-noise and other problems.

Jim Barnes and I presented a paper at that conference. Jim had developed a special auto-correlation function that was convergent for flicker noise, and he demonstrated that by using the second difference of phase he had solved the problem basically of how to deal with flicker noise. This paper was very well received. A lot of excellent comments were made at the conference around Jim's presentation.

The next year, Jim did his PhD thesis using these concepts, and I started on my Master's thesis the same year to try and understand how the classical variance changed as a function of these different noise processes that we see in clocks. Both Jim's and my theses were published in a special issue of the Proceedings of the IEEE in February of 1966. That issue has a lot of really useful papers in it. Jim's and my paper were privileged to be a part of that set.

To see the value of the approach of time domain analysis, an analogy with a rainbow gives a good feel. You have a full spectrum of rainbow colors. Observing the tau dependence of a clock allows you to know all of its different colors of noise, which you get from a sigma-tau diagram. Whereas if you compute a standard deviation or classical variance analysis of the data, all you end up with is a black and white picture. You get intensity, but you don't know the spectrum.

A sigma-tau diagram allows us to determine both the level and the kind (color) of the noise. It also allows us to do optimum estimation, smoothing, and prediction. Once you know what's going on with the clocks, then you can do algorithms to do estimation and optimal smoothing of the data. For GPS, that's extremely important because they go across a tracking station in twelve-hour orbits, and they have to predict time forward several hours before they cross another tracking station, and to get another update for the on-board clock parameters. So predicting forward is extremely important for GPS. The two-sample variance has been a powerful tool.

The other thing it allows you to do, once you know the characteristics of a system, then you do optimum instrumentation. You can develop better GPS receivers. You can develop better measuring systems for your clock. You want to make sure the measurement noise is better than the clock noise, for example.