Reprints from the Early Days of Information Sciences

Reminiscences of the Early Work in DCT

Interview with K.R. Rao

Radomir S. Stanković, Jaakko T. Astola

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Radomir S. Stanković, Jaakko T. Astola, (eds.)

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Preface

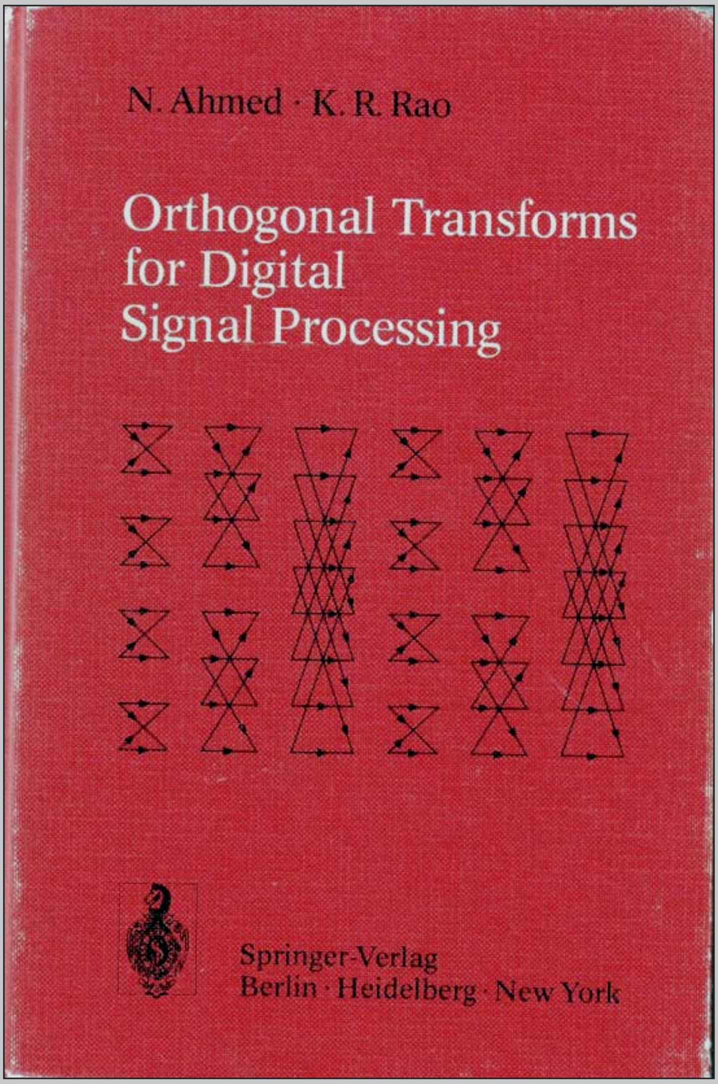
The first book about spectral techniques that I studied in 1976 while preparing my BSc thesis on Walsh and Haar transforms was the book

Ahmed, N., Rao, K.R., *Orthogonal Transforms for Digital Signal Processing,* Springer, Heidelberg, 1975.

Therefore, I was very pleased when I met Prof. K.R. Rao in 2009 at the conference TELSIKS organized at the Faculty of Electronic Engineering in Niš, Serbia. I felt very proud when I got his signature on a sample of the Russian edition of this book.

In September 2011, I met Prof. Rao again at the same conference, and while talking with him I learnt that he gave a tutorial in Tampere, Finland, at the occasion of another conference organized by the **Tampere International Center for Signal Processing,** in 2000. I used the opportunity to ask Prof. Rao for an interview and his reminiscences of the early work in the Discrete Cosine Transform (DCT). This booklet is a short record of this talk with Prof. Rao.

Radomir Stanković


N. Ahmed · K. R. Rao
Orthogonal Transforms for Digital Signal Processing
Springer-Verlag
Berlin · Heidelberg · New York

Contents

Organisation of the Book

The organisation of the presentation in this book was determined by the sequence of events that lead to its preparing.

In Chapter 1, we present the interview with Professor K.R. Rao. While talking he mentioned that Professor Nasir Ahmed pioneered the Discrete cosine transform and was an instrumental for coming up with this idea. We contacted Professor Ahmed and asked for his reminiscences of that work. Thanks to him, we received the reprinted paper on the origins of DCT. Professor Ahmed also helped in making the contact with Dr. T. Raj Natarajan. This determined the contents of the Chapter 2, where the initial paper on DCT is reprinted, and also the contents of the first three sections of the Chapter 3. We also included the first page of the paper in 1970 about the BIFORE spectrum, since this is an early joint publication of Professors Ahmed and Rao and introduces an important concept as it was mentioned in the interview.

Professors Roumen Kounthcev and Zoran Bojković participated in the same conference where we met Prof. Rao and we used this opportunity to shortly interview them. These inteviews, illustrated by a few selected pages of their publications make the contents of another two sections of the same chapter. Prof. Rao used to always work with many students. Therefore, the chapter ends with a gallery of photos of his current students.

The Chapter 4 is the gallery of covers and selected pages of books authored, co-authored, or edited by Professor K.R. Rao. The gallery starts with reprints of the cover and several selected pages of the edition in Russian of the book N. Ahmed, K.R. Rao, *Orthogonal Transfoms for Digital Signal Processing*, Springer, 1975. The included front matter pages show the publication date of the book as well as the name of the publisher in the former Soviet Union, the scientific classification, and the Preface to the Russian Edition written by a renowned expert in the field, Professor A.M. Trachtman, who is himself the author of few books and many important publications in signal processing and spectral techniques in particular. The Russian edition contains a chapter with a review of the work in the area by authors from the Soviet Union and a list of 42 selected additional references. We believe this list of references can be an interesting piece of information to be presented to public notice. The gallery of books continues with covers of several books by Prof. Rao and his associates, including translations of the books to foreign languages.

Radomir S. Stanković, Jaakko T. Astola

Interview with Professor K.R. Rao

People working in spectral methods, they all know your name. But usually we simply say K. R. Rao, and not the full name. So in Serbian we will say K R Rao. But could you please tell us about your name, and your origins, where you started from, and so on. We know that you are now for many years in Texas, but is this the place where you were born, or how you started. So, please, tell us about your first years and your primary education, and your remembering of your professors or teachers and colleagues.

Yes, my full name is Kamisetty Ramamohan Rao. To simplify for everybody to follow that professionally I changed the name to K.R. Rao. I was born in Madras and grew up in a small town (Proddatur) grew up there, went to the elementary school, junior high and highschool there and after high school I went to Madras, India that is a city, now it is called Chennai one of the major cities in India where I did my intermediate for 2 years, and after that went to the Engineering college, Guindy which is a four-year college, majoring in electrical engineering.

Our parents are blessed with 11 children (I am the eldest.). They provided all of us with excellent education and helped us with good careers. After that I worked with some private companies in India (Andhra Pradesh one of the states) till 1957, and that is when I left that job and career to come to the United States. I was awarded a Fulbright travel grant, which took care of my travel from India to the United States and back. Then I joined the University of Florida in Gainesville, initially. I was majoring in power systems at that time, and there was not a very good program there, so they moved me to University of Wisconsin, Madison, Wisconsin. I was there for one semester and I was short of funds so I went back to University of Florida where they offered me assistantship. Before I left for US, I was happily married with one child, and when I left for US in September 1957, we had a 10 month old baby, the son Ramesh. My wife's parents kindly took care of our son Ramesh for 5 years. A year later my wife Karuna joined me, and I did two masters in engineering at the University of Florida, one in electrical engineering, and another in nuclear engineering. At that time my wife went to high school there, in Gainesville, Florida, and in 1960 we moved to University of California at Berkley for the Ph.D program. My wife also went to high school there, in Berkley and also graduated from the high school at that time. Then, my wife was expecting a baby, so we moved after one year in Berkley to Whitworth College in Spokane, Washington as a lecturer. We stayed there for nine months. My wife had the baby, second one is a daughter, Sasirekha and in June of 1962, we went to India with the option of essentially starting a career either in education or industry in India. However after two months in India I went back to USA. My wife was in India along with our two children (Ramesh and Sasirekha) staying with her parents in Hyderabad.They came back to the United States a year later. I started my doctoral program at the University of New Mexico, Albuquerque in 1962.

In 1966, I received my PhD degree in electrical engineering and then we all moved to University of Texas at Arlington to start my educational career. In Arlington, Texas. Both our children went to high school, junior high and high school and then graduated from Arlington high school. My wife went to nursing program and explored number of opportunities/jobs/careers.

From 1966 to till present I have been in the University of Texas at Arlington, going through various promotional changes. We essentially grew up along with the University. The University at that time had only 11 000 students, at the time it started a master's program, and now it has 34 000 students, PhD programs in several areas, lots of well-known faculty, research projects and programs, so forth, various research centers, and a number of buildings, facilities, labs, equipment, etc. It has grown up now to be a major university in the state of Texas, and in fact, in the nation. In 1969 I was promoted as an associate professor and in 1973 as a full professor in the University.

When I was at the University of New Mexico, Albuquerque, I was closely associated with professor Nasir Ahmed and then we both were interested in the Walsh functions, Hadamard functions, applications of Walsh functions, spectral methods, power spectral invariant systems, and the Hadamard transform, Walsh transform, fast algorithms, and the length from, say 4 to 8, 8 to 16, so forth, both forward and inverse Hadamard transforms and that was also called BIFORE, Binary Fourier Representation. We published many papers related to Walsh functions and their applications in signal processing. The results are called Hadamard transform. And then various versions of that, complex Hadamard transform, generalized transforms, generalized Walsh functions and sequency which is similar to the frequency, sequency is related to number of sign changes per the unit interval, and is the basis for sequency spectra.

How you discover this subject? Who introduced it to you?

That was Walsh, I would not say it discovery, we were exposed to Walsh functions, through the publications of Harmuth. We explored that in detail and from that we developed some important properties such as the translation invariant sequency spectra, power spectra, power spectral methods, applications of the power spectra, the Hadamard transform, Fourier transform, in the signal processing arena. Both Dr. Ahmed and I essentially published in well known refereed journals, related to Walsh functions, Hadamard transform, and their related aspects.

In 1974 it was Dr. Ahmed who pioneered the discrete cosine transform. The DCT was published in the IEEE Transactions on Computers as a letter in 1974, its properties, power spectra and then also fast transforms via the well-known discrete Fourier transform.

N. Ahmed, T. Natarajan, and K. R. Rao, "Discrete Cosine Transform", *IEEE Trans. Computers*, 90-93, Jan 1974.

Also Dr. B.G. Lee developed a faster algorithm similar to radix-2 DIF- FFT. That was the fastest algorithm involving real multiplications and additions. After this various versions of the fast discrete transforms tailored for specific architectures/specific platforms have been developed. Later on as things progressed, DCT has been a major component, major function and major operation in the well-known Joint Photographic Experts Group (JPEG). We understand in developing the JPEG, which was both ISO/IEC and ITUT-standard, several different algorithms were proposed. Some using a different transform, prediction-based and so forth, but looking at the overall complexity, performance, visual quality at various levels, bit rates and different color formats, the Standards group chose the DCT-based JPEG standard. In the baseline profile, the DCT is mandatory and that was essentially used, what we now call the JPEG files.

Subsequently, DCT has been utilized in other video encoding standards such as the MPEG 1, MPEG 2, and MPEG 4 for video conferencing, video phone and then, also, some modified versions, like modified DCT, which has been instrumental in developing the so- called MP3 audio.

W.-H. Chen, C. H. Smith, and S. Fralick, "A Fast Computational Algorithm for the Discrete Cosine Transform", *IEEE Trans. Communications*, 25, 1004-1009, Sep 1977.

B. G. Lee, "A new algorithm to compute the discrete cosine transform", *IEEE Trans. Signal Processing*, Vol. ASSP-32, 1243-1245, 1984.

At that time we felt that we had a publication but of course that development received lots ofinterests from the research, professional and academic communities, and that is when we realized it was an important milestone in the general area of discrete transforms in particular and in general in the signal processing field. So, from then on we looked at other related transforms such as the complex Hadamard transform, generalized discrete transforms, including but not limited to the Discrete Fourier transform. Multidimensional DCT, power spectra, convolution, correlation theorems dealing with the DCT, inverse DCT, fast algorithms both for forward and inverse, and as well what is called the scalable properties from 4 to 8, 8 to 16, and 16 to 32.

Dr. Nasir Ahmed was essentially instrumental in coming up with the idea of the discrete cosine transform, and that was when we contributed to its development and its deployment in the industry. Applications especially in data compression, bandwidth reduction and bit rate reduction at that time adopted by the company Compression Labs which was subsequently acquired by VTEL.

Compression Labs was the only one which tried the DCT in the video encoding arena and Dr. Chen developed the fast algorithm for the DCT.

N. Ahmed, "How I came up with the Discrete Cosine Transform", *Digital Signal Processing*, Vol. 1, No. 1, 1991, 4-5.

Subsequently other versions of the DCT like integer DCT, integer modified DCT, directional DCT and so forth, have been utilized in other standards such as the MPEG 4, part 10 which is also called H.264 advanced video coding developed for the JVT (Joint Video Team). In that, DCTs of size 4 by 4 or 8 by 8, have been utilized. In the MPEG 4 part 2 video where there are video object plane, scalable coding and so forth the standard 8 by 8 DCT has been utilized. More recently now larger size integer DCTs including 16 by 16 and 32 by 32 are essentially being looked into in high efficiency video coding, next generation video coding. This is supposed to lead to what they call initially the head start to H.265 (not official). It has a number of additional operations, such as mode dependent directional transforms, directional motion estimation, motion compensation, and large size motion estimation blocks, motion vectors aimed at applications such as ultra high definition TV, digital cinema, super resolution high definition television, and so forth. This was developed as a committee draft in March 2011 and now is in various stages, working draft, final working draft, draft international standard, final draft international standard, and finally leading to international standard. This is supposed to reach the international standard of United States towards the end of 2012, or early 2013. The importance is in again choosing the integer version of large size DCTs as a major functional block in the overall encoder and of course the inverse operations, inverse integer DCTs in the decoder. One special advantage of the integer DCT is the integer arithmetic, therefore, there is no, what we call drift or a mismatch in both forward and backward integer DCTs, which was of some problem or concern in the standard DCTs. DCT was implemented in the DSP chip, in software, and later on many companies developed DSP chips dedicated for DCT, and as the technology and the logic cycles, chip development storage capabilities so forth have increased, and went into submicron technology, then, the entire, encoder/decoder, video, audio, multiplexing, control signals, signaling, video from acquisition in analog to digital format and format as luminance and color difference and different formats such as the 444, 420, 421,411. Then going through the encoder, all the functions involved in the encoder such as the transform, motion estimation, motion compensation and prediction, transform, quantization, perceptual weighting and then the entropy encoding, variable encoding of this transform functions various order, motion estimation, motion vectors, different overhead, controlling factors, and then all the decoding operations, final inverse operation to and from different color formats YIQ, YUV to RGB and then finally for display purposes.

Now all this can be implemented even at HDTV resolution in one single chip (encoder/decoder). We are very happy that the discrete cosine transform DCT and its various versions, modified DCT, integer DCT, modified integer DCT, directional DCT, all of this at different sizes, forward transform, inverse transform, fast algorithms, and also multidimensionalDCTs, and then 4, 8, 16, 32 size DCTs, all of this have contributed significantly to the overall video, audio and images compression/coding, yes, what we now call the multimedia communications devices. So we are happy to see this. We hope there will be further developments, and further applications to this very challenging and interesting field.

There are also, other transforms like the complex Hadamard transform, it has also found applications in the general signal processing arena and our colleagues, professor Roumen Kounchev and professor Romiana Koucheva, they can also give their various contributions, in the general field of complex Hadamard transform and their applications in the signal processing arena.

Can you, please, tell us how it was organized the work with Professor Ahmed? How you used to work, do you work during the early morning or late at night, how it was this research done, did you use much computers or maybe you use ordinary typewriters to prepare your book and such things, how it was at that time?

Professor Ahmed was already there at the University of New Mexico, Albuquerque, when I joined. He studied the masters and then he was starting on his PhD. So we were together for four years in New Mexico, Albuquerque, that is when we worked together. We worked both late in the night and again early mornings and that is when we developed mutual interests in the Walsh functions, Walsh transform, BIFORE transform, complex Hadamard transform and as well as the generalized discrete transforms, their fast algorithms and basically their applications. At that time we were using what you call the IBM machines and the IBM computer cards to write the programs mostly at that time in Fortran 77.

Fortran 77 was the most popular one. So we had to wait till the next day to receive our IBM cards and submit the program using the IBM cards. So at that time card machines, card punching operations were very popular. Also, we used the IBM electric machines for typing. In fact, to prepare my presentations, both my wife and our children helped me using the IBM electric to type the papers, dissertation etc. They spent all nights helping me with that.

Professor Ahmed for the first time was exposed to Walsh functions when he went to work for Honeywell, where he met and worked with Ferd Ohnsorg, who had developed the BIFORE transform. Professor Ahmed joined Honeywell in January 1966 and left for Kansas State around July of 1968. As such, we initiated our collaboration in this area during this time frame, and we used to regularly attend the annual Walsh Functions symposia organized by Prof. Harmuth at the Catholic Univ. in Washington, D.C.

In January 1966 Professor Ahmed graduated from the University of New Mexico (UNM). I received the Ph.D from UNM in June 1966 and immediately moved with my family to University of Texas, Arlington. At that time it was still called Arlington State College. Then in September 1966, they changed the name to University of Texas at Arlington and started the masters program in three areas in engineering.

Ohnsorg, F.R., "Binary Fourier representation", 1966 Spectrum Analysis Techniques Symposium, Hooneywell Research Center, Hopkins, Minnesota, USA.

Could you, please, tell us how was the cooperation with other colleagues working in the field of Walsh functions at the beginning? Did you meet Professor Harmuth or did you use to meet each other at conferences?

Yes, Professor Harmuth. I met him several times, mainly in Washington DC, that is when he used to organize every year Walsh functions conference. We (Dr. Ahmed and I) used to attend those conferences and present papers there. Also we used to meet and discuss with Professor Harmuth who gave us many ideas, many solutions in developing the Walsh functions and their applications in different fields. So he was our inspiration. He was our guide, and he was our what we can call, the source for developing the various Walsh functions and applications.

How about cooperation with researchers in India? Did you keep the links there, or are there workings in this field?

Well, the researchers in India, I myself have connected with them through workshops, tutorials, in many countries, now more than 100, including India, and then there were other researchers who used the Walsh functions, Walsh Hadamard functions, BIFORE transform, and its various versions, again, the general signal processing arena. They directed a number of theses, the dissertations. Among the faculty, the most notable one I canhtink of is Dr. H. B. Kekre from NT, Bombay. Our publications, tutorials, workshops, seminars, invited papers, keynote speeches, have received wide attention, and also have attracted the researchers in this field.

Did you have links with research in Soviet Union, East Europe at that time? Did you had any access or it was not so easy to get literature that was published there?

No, in Soviet Union the access/communications were difficult. Similarly in Eastern Europe. Except, the research/academic community in the Eastern Europe, they had access to all of our publications. They utilized the publications to further the field and also to extend the applications.

How it is now? Do you have cooperation with East Europe now? I am really glad that you came here in Niš, Serbia, and not for the first time.

Well yes, Professor Bojković came about 4-5 times to University of Texas, Arlington, as a visiting professor, visiting scholar, staying as you know for different periods. I also visited him in Serbia about three times. He is highly motivated and dedicated to research.

Remark by Prof. Bojković
Thirty years ago I started with Arlington, thirty years ago....

Thirty years. Yes. So we were very fortunate to have professor Bojković and also for his contributions to this field and then because of him we have about 4 books, including one in multimedia communications, which have received extensive coverage and also, Indian editions, and paperback editions. I still hope to have this cooperation with professor Bojković and his doctoral students, such as Milovanović. Similarly, Dr. Vladimir Britanak from the Academy of Sciences in Slovakia, he also visited my lab. We published several papers related to modified DCT, modified discrete sine transform and the various versions of that, a number of fast algorithms. We also have published a book Dr. Vladimir Britanak from Slovakia, Dr. Patrick Yip from Canada and I are the three authors of the book.

Reprint

N. Ahmed, T. Natarajan, K.R. Rao, ”Discrete Cosine Transform”
IEEE Trans. Computers, January 1974, 90-93.

First page of
N. Ahmed, K.R. Rao, ”Spectral Analysis of
Linear Digital Systems using BIFORE”
Electronic Letters, 2, 1970, 43-44.

If only the L values R_0 through R_{L-1} are required, then it can be seen by comparing (15) and (11) that the new algorithm will require fewer multiplications than the FFT method if

$$L < 11.2\,(1 + \log_2 N) \tag{16a}$$

and will require fewer additions if

$$L < 5.6\,(1 + \log_2 N). \tag{16b}$$

Therefore, we conclude that the new algorithm will generally be more efficient than the FFT method if

$$N \leq 128 \tag{17a}$$

or

$$L \leq 10(1 + \log_2 N). \tag{17b}$$

Conclusion

A new algorithm for computing the correlation of a block of sampled data has been presented. It is a direct method which trades an increased number of additions for a decreased number of multiplications. For applications where the "cost" (e.g., the time) of a multiplication is greater than that of an addition, the new algorithm is always more computationally efficient than direct evaluation of the correlation, and it is generally more efficient than FFT methods for processing 128 or fewer data points, or for calculating only the first L "lags" for $L \leq 10 \log_2 2N$.

References

[1] S. Winograd, "A new algorithm for inner product," *IEEE Trans. Comput.* (Corresp.), vol. C-17, pp. 693–694, July 1968.

Discrete Cosine Transform

N. AHMED, T. NATARAJAN, AND K. R. RAO

***Abstract*—A discrete cosine transform (DCT) is defined and an algorithm to compute it using the fast Fourier transform is developed. It is shown that the discrete cosine transform can be used in the area of digital processing for the purposes of pattern recognition and Wiener filtering. Its performance is compared with that of a class of orthogonal transforms and is found to compare closely to that of the Karhunen–Loève transform, which is known to be optimal. The performances of the Karhunen–Loève and discrete cosine transforms are also found to compare closely with respect to the rate-distortion criterion.**

***Index Terms*—Discrete cosine transform, discrete Fourier transform, feature selection, Haar transform, Karhunen–Loève transform, rate distortion, Walsh–Hadamard transform, Wiener vector and scalar filtering.**

Introduction

In recent years there has been an increasing interest with respect to using a class of orthogonal transforms in the general area of digital signal processing. This correspondence addresses itself towards two problems associated with image processing, namely, pattern recognition [1] and Wiener filtering [2].

In pattern recognition, orthogonal transforms enable a noninvertible transformation from the pattern space to a reduced dimensionality feature space. This allows a classification scheme to be implemented with substantially less features, with only a small increase in classification error.

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In discrete Wiener filtering applications, the filter is represented by an $(M \times M)$ matrix G. The estimate $\hat{X}$ of data vector X is given by GZ, where $Z = X + N$ and N is the noise vector. This implies that approximately $2M^2$ arithmetic operations are required to compute $\hat{X}$. Use of orthogonal transforms yields a G in which a substantial number of elements are relatively small in magnitude, and hence can be set equal to zero. Thus a significant reduction in computation load is realized at the expense of a small increase in the mean-square estimation error.

The Walsh–Hadamard transform (WHT), discrete Fourier transform (DFT), the Haar transform (HT), and the slant transform (ST), have been considered for various applications [1], [2], [4]–[9] since these are orthogonal transforms that can be computed using fast algorithms. The performance of these transforms is generally compared with that of the Karhunen–Loève transform (KLT) which is known to be optimal with respect to the following performance measures: variance distribution [1], estimation using the mean-square error criterion [2], [4], and the rate-distortion function [5]. Although the KLT is optimal, there is no general algorithm that enables its fast computation [1].

In this correspondence, a discrete cosine transform (DCT) is introduced along with an algorithm that enables its fast computation. It is shown that the performance of the DCT compares more closely to that of the KLT relative to the performances of the DFT, WHT, and HT.

Discrete Cosine Transform

The DCT of a data sequence $X(m)$, $m = 0, 1, \cdots, (M-1)$ is defined as

$$G_x(0) = \frac{\sqrt{2}}{M}\sum_{m=0}^{M-1} X(m)$$

$$G_x(k) = \frac{2}{M}\sum_{m=0}^{M-1} X(m)\cos\frac{(2m+1)k\pi}{2M}, \qquad k = 1, 2, \cdots, (M-1) \tag{1}$$

where $G_x(k)$ is the kth DCT coefficient. It is worthwhile noting that the set of basis vectors $\{1/\sqrt{2}, \cos((2m+1)k\pi)/(2M)\}$ is actually a class of discrete Chebyshev polynomials. This can be seen by recalling that Chebyshev polynomials can be defined as [3]

$$\hat{T}_0(\xi_p) = \frac{1}{\sqrt{2}}$$

$$\hat{T}_k(\xi_p) = \cos(k\cos^{-1}\xi_p), \qquad k, p = 1, 2, \cdots, M \tag{2}$$

where $\hat{T}_k(\xi_p)$ is the kth Chebyshev polynomial.

Now, in (2), ξ_p is chosen to be the pth zero of $\hat{T}_M(\xi)$, which is given by [3]

$$\xi_p = \cos\frac{(2p-1)\pi}{2M}, \qquad p = 1, 2, \cdots, M. \tag{3}$$

Substituting (3) in (2), one obtains the set of Chebyshev polynomials

$$\hat{T}_0(p) = \frac{1}{\sqrt{2}}$$

$$\hat{T}_k(p) = \cos\frac{(2p-1)k\pi}{2M}, \qquad k, p = 1, 2, \cdots, M. \tag{4}$$

From (4) it follows that the $\hat{T}_k(p)$ can equivalently be defined as

$$T_0(m) = \frac{1}{\sqrt{2}}$$

$$T_k(m) = \cos\frac{(2m+1)k\pi}{2M}; \qquad k = 1, 2, \cdots, (M-1),$$

$$m = 0, 1, \cdots, M-1. \tag{5}$$

Comparing (5) with (1) we conclude that the basis member cos ((2m +

$1)k\Pi)/(2M)$ is the kth Chebyshev polynomial $T_k(\xi)$ evaluated at the mth zero of $T_M(\xi)$.

Again, the inverse cosine discrete transform (ICDT) is defined as

$$X(m) = \frac{1}{\sqrt{2}} G_x(0) + \sum_{k=1}^{M-1} G_x(k) \cos \frac{(2m+1)k\Pi}{2M}, \quad m = 0, 1, \cdots, (M-1). \qquad (6)$$

We note that applying the orthogonal property [3]

$$\sum_{m=0}^{M-1} T_k(m)\, T_l(m) = \begin{cases} M/2, & k = l = 0 \\ M/2, & k = l \neq 0 \\ 0, & k \neq l \end{cases} \qquad (7)$$

to (6) yields the DCT in (1).

If (6) is written in matrix form and Λ is the $(M \times M)$ matrix that denotes the cosine transformation, then the orthogonal property can be expressed as

$$\Lambda^T \Lambda = \frac{M}{2} [I] \qquad (8)$$

where Λ^T is the transpose of Λ and $[I]$ is the $(M \times M)$ identity matrix.

Motivation

The motivation for defining the DCT is that it can be demonstrated that its basis set provides a good approximation to the eigenvectors of the class of Toeplitz matrices defined as

$$\psi = \begin{bmatrix} 1 & \rho & \rho^2 & & & \rho^{M-1} \\ \rho & 1 & \rho & \cdot & \cdot\ \cdot & \rho^{M-2} \\ \cdot & & & \cdot & & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & & \cdot\ \cdot \\ \rho^{M-1} & \rho^{M-2} & & \cdot & \cdot\ \cdot & 1 \end{bmatrix}, \quad 0 < \rho < 1. \qquad (9)$$

For the purposes of illustration, the eigenvectors of ψ for $M = 8$ and $\rho = 0.9$ are plotted (see Fig. 1) against

$$\left\{ \frac{1}{\sqrt{2}}, \cos \frac{(2m+1)k\Pi}{16}, \quad k = 1, 2, \cdots, 7, m = 0, 1, \cdots, 7 \right\} \qquad (10)$$

which constitute the basis set for the DCT. The close resemblance (aside from the 180° phase shift) between the eigenvectors and the set defined in (10) is apparent.

Algorithms

It can be shown that (1) can be expressed as

$$G_x(0) = \frac{\sqrt{2}}{M} \sum_{m=0}^{M-1} X(m)$$

$$G_x(k) = \frac{2}{M} \operatorname{Re} \left\{ e^{(-ik\Pi)/(2M)} \sum_{m=0}^{2M-1} X(m)\, W^{km} \right\}, \quad k = 1, 2, \cdots, (M-1) \qquad (11)$$

where

$$W = e^{-i2\Pi/2M}, \quad i = \sqrt{-1},$$

$$X(m) = 0, \quad m = M, (M+1), \cdots, (2M-1)$$

and Re $\{\cdot\}$ implies the real part of the term enclosed. From (11) it follows that all the M DCT coefficients can be computed using a $2M$-point fast Fourier transform (FFT). Since (1) and (6) are of the same form, FFT can also be used to compute the IDCT. Similarly, if a discrete sine transform were defined, then the Re $\{\cdot\}$ in (11) would be replaced by Im $\{\cdot\}$, which denotes the imaginary part of the term enclosed.

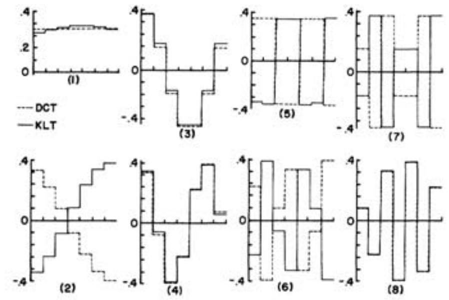

Fig. 1. Eigenvectors of (8 × 8) Toeplitz matrix (ρ = 0.9) and basis vectors of the DCT.

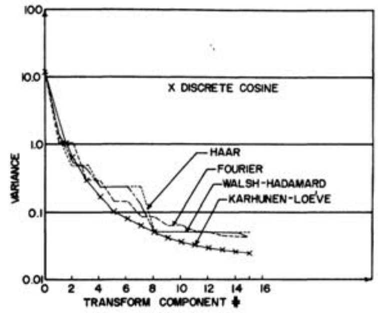

Fig. 2. Transform domain variance; $M = 16$, $\rho = 0.95$.

Computational Results

In image processing applications, ψ in (9) provides a useful model for the data covariance matrix corresponding to the rows and columns of an image matrix [6], [7]. The covariance matrix in the transform domain is denoted by Ψ and is given by

$$\Psi = \Lambda\, \psi\, \Lambda^{*T} \qquad (12)$$

where Λ is the matrix representation of an orthogonal transformation and Λ^* is the complex conjugate of Λ. From (12) it follows that Ψ can be computed as a two-dimensional transform of ψ.

Feature Selection

A criterion for eliminating features (i.e., components of a transform vector), which are least useful for classification purposes, was developed by Andrews [1]. It states that features whose variances (i.e., main diagonal elements of Ψ) are relatively large should be retained. (Fig. 2 should be retained.) Fig. 2 shows the various variances ranked in decreasing order of magnitude. From the information in Fig. 2 it is apparent that relative to the set of orthogonal transforms shown, the DCT compares most closely to the KLT.

Wiener Filtering

The role of orthogonal transforms played in filtering applications is illustrated in Fig. 3 [2]. Z is an $(M \times 1)$ vector which is the sum of a

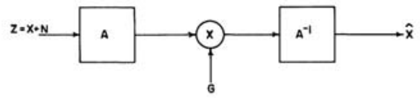

Fig. 3. Wiener filtering model.

TABLE I
MEAN-SQUARE ERROR PEFORMANCE OF VARIOUS TRANSFORMS FOR SCALAR WIENER FILTERING; $\rho = 0.9$

Transform \ M	2	4	8	16	32	64
Karhunen–Loève	0.3730	0.2915	0.2533	0.2356	0.2268	0.2224
Discrete cosine	0.3730	0.2920	0.2546	0.2374	0.2282	0.2232
Discrete Fourier	0.3730	0.2964	0.2706	0.2592	0.2441	0.2320
Walsh–Hadamard	0.3730	0.2942	0.2649	0.2582	0.2582	0.2559
Haar	0.3730	0.2942	0.2650	0.2589	0.2582	0.2581

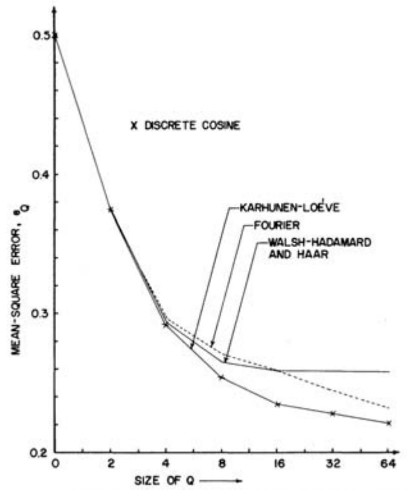

Fig. 4. Mean-square error performance of various transforms for scalar Wiener filtering; $\rho = 0.9$.

vector X and a noise vector N. X is considered to belong to a random process whose covariance matrix is given by ψ which is defined in (9). The Wiener filter G is in the form of an $(M \times M)$ matrix. A and A^{-1} represent an orthonormal transform and its inverse, respectively, while $\hat{X}$ denotes the estimate of X, using the mean-square error criterion.

We restrict our attention to the case when G is constrained to be a diagonal matrix Q. This class of Wiener filters is referred to as *scalar* filters while the more general class (denoted by G) is referred to as vector filters. The additive noise (see Fig. 3) is considered to be white, zero mean, and uncorrelated with the data. If the mean-square estimation error due to scalar filtering is denoted by e_Q, then e_Q can be expressed as [4]

$$e_Q = 1 - \frac{1}{M}\sum_{s=1}^{M} \frac{\Psi_x^2(s,s)}{\Psi_x(s,s) + \Psi_n(s,s)} \tag{13}$$

where Ψ_x and Ψ_n denote the transform domain covariance matrices of the data and noise, respectively. Table I lists the values of e_Q for different values of M for the case $\rho = 0.9$ and a signal-to-noise ratio of unity. From Table I it is evident that the DCT comes closest to the KLT which is optimal. This information is presented in terms of a set of performance curves in Fig. 4.

ADDITIONAL CONSIDERATIONS

In conclusion, we compare the performance of the DCT with KLT, DFT, and the identity transforms, using the rate-distortion criterion [5]. This performance criterion provides a measure of the information rate R that can be achieved while still maintaining a fixed distortion D, for encoding purposes. Considering Gaussian sources along with the mean-square error criterion, the rate-distortion performance measure is given by [5]

$$R(A, D) = \frac{1}{2M}\sum_{j=1}^{M} \max\left\{0, \ln\left(\frac{\sigma_j}{\theta}\right)\right\} \tag{14a}$$

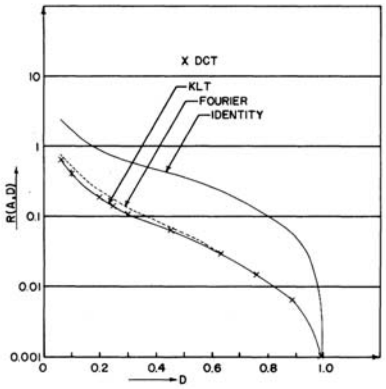

Fig. 5. Rate versus distortion for $M = 16$ and $\rho = 0.9$.

$$D = \frac{1}{M} \sum_{j=1}^{M} \min(\theta, \sigma_j) \tag{14b}$$

where A denotes the orthogonal transformation and the σ_j are the main diagonal terms of the transform domain covariance matrix Ψ in (12).

The rate-distortion pertaining to $M = 16$ and $\rho = 0.9$ is shown in Fig. 5, from which it is evident that the KLT and DCT compare more closely than the KLT and DFT.

Summary

It has been shown that the DCT can be used in the area of image processing for the purposes of feature selection in pattern recognition; and scalar-type Wiener filtering. Its performance compares closely with that of the KLT, which is considered to be optimal. The performances of the KLT and DCT are also found to compare closely, with respect to the rate-distortion criterion.

Acknowledgment

The authors wish to thank Dr. H. C. Andrews, University of Southern California, for his valuable suggestions pertaining to the rate-distortion computation.

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Minimization of Linear Sequential Machines

CHI-TSONG CHEN

***Abstract*—An algorithm is presented to minimize linear sequential machines to reduced form.**

***Index Terms*—Algorithm, linear machine, minimization, reduced machine, reduction, sequential machine.**

I. Introduction

The minimization of a linear sequential machine to a reduced form is an important topic and is discussed in many texts [1]–[4]. The minimization procedure presented in [1]–[4] is as follows. Let $\{A, B, C, D\}$ be an n-dimensional linear machine over $GF(p)$, and let r, with $r < n$, be the rank of the diagnostic matrix $K \triangleq [C'A'C' \cdots (A')^{n-1}C']'$, where the prime stands for the transpose. Define an $r \times n$ matrix T consisting of the first r linearly independent rows of K, and an $n \times r$ matrix R denoting the right inverse of T so that $TR = I$. Then the linear machine $\{TAR, TB, CR, D\}$ is a reduced form of $\{A, B, C, D\}$. In this correspondence, an algorithm will be introduced to find a special set of r linearly independent rows in K. A reduced machine can then be read out from this algorithm without the need of inverting any matrix. Furthermore, the reduced machine will be in a canonical form.

II. Algorithm

Let C be a $q \times n$ matrix, and let c_i be the ith row of C. The diagnostic matrix of $\{A, B, C, D\}$ will be arranged in the following order:

$$P = \begin{bmatrix} c_1 \\ c_1A \\ \vdots \\ c_1A^{n-1} \\ \vdots \\ c_q \\ c_qA \\ \vdots \\ c_qA^{n-1} \end{bmatrix}. \tag{1}$$

Now an algorithm will be introduced to find the first r linearly independent rows in P. This is achieved by a series of elementary transformations. Let

$$K_rK_{r-1} \cdots K_2K_1P \triangleq KP \tag{2}$$

where K_i are lower triangular matrices with all diagonal elements unity, and are obtained in the following manner. Let $p_1(j)$ be the first nonzero element from the left in the first row of P, where j denotes the position. Then K_1 is chosen so that all, except the first, elements of the jth column of K_1P are zero. Let $p_2(l)$ be the first nonzero element from the left of the second row of K_1P. Then K_2 is chosen so that all, except the first two, elements of the lth column of K_2K_1P are zero. Proceed in this manner until all linearly independent rows of P are found. Note that in this process, if one row is identically zero, then proceed to the next nonzero row. By multiplying these K_i, we obtain $K = K_rK_{r-1} \cdots K_2K_1$. Note that KP will be finally of form

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SPECTRAL ANALYSIS OF LINEAR DIGITAL SYSTEMS USING BIFORE*

Indexing terms: Digital systems, Fourier transforms

A spectral analysis of linear digital systems using BIFORE is carried out. Expressions for power and amplitude spectra of the input and output signals are developed. The frequency-sequency structure of the power spectra, which are invariant to cyclic shift of the sampled data, is analysed.

Introduction: The object of this letter is to carry out a spectral analysis of linear digital systems using BInary FOurier REpresentation (BIFORE) and to analyse the frequency-sequency structure of the BIFORE power spectra.

BIFORE *or Hadamard transform:* BIFORE or Hadamard transform[1, 2] (b.t. or h.t.) of an n-periodic sampled data

$$\{x(k)\}' = \{x(0), x(1), x(2), \ldots, x(n-1)\}$$

where prime denotes transpose of a matrix and $k = \log_2 n$ is

$$\{B_x(k)\} = \frac{1}{n}[H(k)]\{x(k)\} \quad \ldots \quad (1)$$

$\{B_x(k)\} = \{B_x(0), B_x(1), \ldots, B_x(n-1)\}$ and $H(k)$ is an $n \times n$ Hadamard matrix. The signal can be recovered uniquely by the inverse b.t.; i.e.

$$\{x(k)\} = [H(k)]\{B_x(k)\} \quad \ldots \quad (2)$$

$HH' = nI(k)$ where $I(k)$ is an $n \times n$ unit matrix. Analogous to the fast Fourier transform[3] (f.f.t.), fast BIFORE transform[4, 5] (f.b.t. or f.h.t.) has also been developed.

Using the shift theorem, Ohnsorg[6] developed a power spectrum for the BIFORE system which is invariant to cyclic shifts of the sampled data. The BIFORE point spectrum $\{P_m\}$ of $\{x(k)\}$ is defined as

$$P_0 = B_x^2(0)$$

$$P_m = \sum_{i=2^{m-1}}^{2^m-1} B_x^2(i) \qquad m = 1, 2, 3, \ldots, k \quad \ldots \quad (3)$$

The BIFORE spectrum consists of $k+1$ points, in contrast to the Fourier spectrum which has $n/2+1$ independent spectrum points. The BIFORE spectrum provides a means of compressing data and yet retains some information pertaining to its harmonic content, which follows.

Frequency structure of power spectra: When $n = 8$, the power spectra are

$$P_0 = B_x^2(0) \quad P_1 = B_x^2(2) + B_x^2(3),$$

$$P_3 = B_x^2(4) + \ldots + B_x^2 \qquad (7)$$

An inspection of Walsh functions, sampled data and Fourier harmonics (Fig. 1) gives the following frequency–sequency structure.

BIFORE component	Frequency	Sequency
$B_x(0)$	0	0
$B_x(1)$	4	7
$B_x(2)$	2	3
$B_x(3)$	2	4
$B_x(4)$	1	1
$B_x(5)$	3	6
$B_x(6)$	1	2
$B_x(7)$	3	5

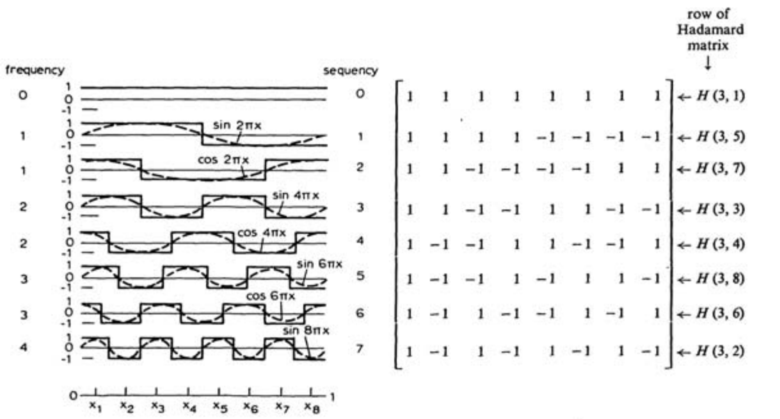
Denote $F(P_i)$ as the frequency content of spectral point P_i. Then

$$F(P_0) = 0 \quad F(P_3) = 1, 3 \quad F(P_2) = 2 \quad F(P_1) = 4$$

or, in general, for any n where $n = 2^k$

$$\left.\begin{aligned} &F(P_0) = 0 \\ &F(P_k) = 1, 3, 5, \ldots, \left(\frac{n}{2} - 1\right) \\ &F(P_{k-1}) = 2, 6, 10, \ldots, \left(\frac{n}{2} - 2\right) \\ &F(P_{k-2}) = 4, 12, 20, \ldots, \left(\frac{n}{2} - 4\right) \\ &\ldots \\ &F(P_1) = \frac{n}{2} \end{aligned}\right\} \quad \ldots \quad (4)$$

Thus each spectral point represents the power content of a group of frequencies rather than that of a single frequency as in the case of Fourier spectra. The frequency grouping, however, is not arbitrary. From eqn. 4 one can observe that each group contains a fundamental and the set of all odd harmonics relative to that fundamental. This corresponds



Walsh functions and Fourier harmonics — Sampled data of Walsh functions

Fig. 1 *Fourier harmonics, Walsh functions and Hadamard matrices for n = 8*

* A paper based on some of this material was presented at the 3rd Asilomar Conference on Circuits and Systems, Pacific Grove, Calif., USA, December 1969

Associates of Professor K.R. Rao

Nasir Ahmed

EDUCATION
Ph.D., University of New Mexico, Albuquerque, NM, 1966
M.S., University of New Mexico, Albuquerque, NM, 1963
B.S., University College of Engineering, Bangalore, India, 1961

EXPERIENCE
1/01/01-present: Retired (Emeritus) Presidential Professor of Electrical & Computer Engineering, University of New Mexico 1/17/96 -12/31/00
Presidential Professor and Associate Provost for Research and Dean of Graduate Studies, University of New Mexico (UNM):

HONORARY AND PROFESSIONAL SOCIETIES

Fellow, Institute of Electrical and Electronic Engineers, since 1985, *for contributions to digital signal processing and engineering education.*

Member, American Society of Engineering Education.

Member, Sigma Xi.

FUNDAMENTAL RESEARCH CONTRIBUTION

Played a pioneering role in the development of orthogonal transforms for compressing video data. In particular, Ahmed was the leading author of the ***discrete cosine transform (DCT),*** which was published in the *IEEE Transactions of Computers* in 1974. The DCT is now the most widely used transform in numerous *research and commercial applications* (e.g., high-definition digital television, videophones, teleconferencing, image transmission via the internet, multimedia systems).

DIGITAL SIGNAL PROCESSING 1, 4–5 (1991)

How I Came Up with the Discrete Cosine Transform

Nasir Ahmed

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During the late sixties and early seventies, there was a great deal of research activity related to digital orthogonal transforms and their use for image data compression. As such, there were a large number of transforms being introduced with claims of better performance relative to others transforms. Such comparisons were typically made on a *qualitative* basis, by viewing a set of "standard" images that had been subjected to data compression using transform coding techniques. At the same time, a number of researchers were doing some excellent work on making comparisons on a *quantitative* basis. In particular, researchers at the University of Southern California's Image Processing Institute (Bill Pratt, Harry Andrews, Ali Habibi, and others) and the University of California at Los Angeles (Judea Pearl) played a key role. In this regard, the *variance criterion* and the *rate distortion criterion* were developed and used extensively as performance measures for evaluating image data compression. In addition, the Karhunen–Loeve transform (KLT) evolved as the optimal transform for comparison purposes. With this as background, I can now address the DCT issue.

What intrigued me was that the KLT was indeed the optimal transform on the basis of the mean-square-error criterion and the first-order Markov process model, and yet there was no efficient algorithm available to compute it. As such, the focus of my research was to determine whether it would be possible to come up with a good approximation to the KLT that could be computed efficiently. An approach that I thought might be worth looking into was *Chebyshev interpolation*, a neat discussion of which was available in a text book (*Computer Evaluation of Mathematical Functions*, by C. T. Fike, Prentice-Hall, 1968, Sections 7.4 and 7.5). This was in early 1972, and I wrote a proposal to the National Science Foundation (NSF) to study a "cosine transform" using Chebyshev polynomials of the form

$$T_0(m) = (1/N)^{1/2}, \quad m = 1, 2, \ldots, N$$

$$T_k(m) = (2/N)^{1/2} \cos \frac{(2m-1)k\pi}{2N}, \quad k = 1, 2, \ldots, N.$$

The motivation for looking into such "cosine functions" was that they closely resembled KLT basis functions for a range of values of the correlation coefficient ρ (in the covariance matrix). Further, this range of values for ρ was relevant to image data pertaining to a variety of applications.

Much to my disappointment, NSF did not fund the proposal; I recall one reviewer's comment to the effect that the whole idea seemed "too simple." Hence I decided to work on this problem with my Ph.D. student Mr. T. Natarajan and my friend Dr. Ram Mohan Rao at the University of Texas at Arlington. In fact, I remember dedicating the whole summer of 1973 to work on this problem. The results that we got appeared too good to be true, and I therefore decided to consult Harry Andrews later that year at a conference in New Orleans. We were both invited speakers at a session on Walsh Functions. Harry suggested that I check out the performance of this "cosine transform" using the rate distortion criterion. He then sent me the computer program to do so. The results again showed that this transform performed better than all the others, and its performance compared very closely with that of the KLT. When I sent the results back to Harry Andrews, he suggested that I publish them. As such, I sent them to the *IEEE Computer Transactions*, and the paper was then published in the January 1974 issue. I recall that we decided to send it in as a correspondence item in order to get it published with mini-

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mum delay. Little did we realize at that time that the resulting "DCT" would be widely used in the future! It is indeed gratifying to see that the DCT is now essentially a "standard" in the area of image data compression via transform coding techniques.

NASIR AHMED was born in Bangalore, India, in 1940. He received the B.S. degree in electrical engineering from the University of Mysore, India, in 1961, and the M.S. and Ph.D. degrees from the University of New Mexico in 1963 and 1966, respectively. From 1966 to 1968 he worked as Principal Research Engineer in the area of information processing at the Systems and Research Center, Honeywell, Inc., St. Paul, Minnesota. He was with Kansas State University, Manhattan, from 1968 to 1983. Since 1983 he has been a Professor in the Electrical and Computer Engineering Department at the University of New Mexico, Albuquerque. He became the Chairman of this department in July 1989. In August 1985 he was awarded one of the twelve Presidential Professorships at the University of New Mexico. He is the leading author of *Orthogonal Transforms for Digital Signal Processing* (Springer-Verlag, 1975), and *Discrete-Time Signals and Systems* (Reston, 1983), and coauthor of *Computer Science Fundamentals* (Merrill, 1979). He is also the author of a number of technical papers in the area of digital signal processing. Dr. Ahmed was an Associate Editor for the *IEEE Transactions on Acoustics, Speech, and Signal Processing* (1982–1984) and is currently an Associate Editor for the *IEEE Transactions on Electromagnetic Compatibility* (Walsh Functions Applications).

Publications of Professor Nasir Ahmed with Dr. T. Natarajan

A. Book

N. Ahmed & T. Natarajan, *Discrete-Time Signals and Systems,* Reston, (a subsidiary of Prentice Hall), Reston, VA, 1983; translated into Japanese.

B. Refereed Journals and Conference Papers/Presentations

1. N. Ahmed and T. Natarajan, K. R. Rao, "Discrete cosine transform", *IEEE Trans. Computers,* Jan. 1974, 90-93.

2. N. Ahmed and T. Natarajan, "On logical and arithmetic autocorrelation functions", *IEEE Trans. Electromag. Compat.,* Aug. 1974, 177-183.

3. N. Ahmed and T. Natarajan, "Interframe transform coding of picture data", *Proc. 9th Asilomar Conf. on Circuits, Systems and Computers,* Nov. 1975.

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6. T. Natarajan and N. Ahmed, "On interframe transform coding", *IEEE Trans. Communications,* Nov. 1977, 1323-1329.

7. T. Natarajan and N. Ahmed, "Performance evaluation for transform coding using a nonseparable covariance model", *IEEE Trans. Communications,* Feb. 1978, 310-312.

Dr. **T. Raj Natarajan** is currently the CEO of Forum Communication Systems, Richardson, TX which he co-founded in 1991. Forum is the leading manufacturer of Audio Conferencing and Emergency Alerting systems. He got his Ph.D degree from Kansas Sate University in 1976. He was the co- founder and VP of Engineering at DSP Technology Corp., Carrollton, TX from 1985 to 1991. His previous experience includes senior positions at Atlantic Richfileld Oil Co., Mead Office Systems, and Texas Instruments. He is the holder of three U.S. patents and is a co- inventor of the DCT transform technique which is part of the international standard for video compression. He is the co-author with Prof. Nasir Ahmed of a text book "Discrete-Time Signals and Systems," published by Reston Publishing.

Collaborators and associates of Prof. K.R. Rao

Prof. **Roumen Kountchev** of Technical University of Sofia, Bulgaria is a collaborator of Prof. K.R. Rao, that was present while we talked with Prof. Rao. We used the opportunity to ask him about his work in Spectral techniques and especially about the joint work with Prof. Rao. It was nice to learn that Prof. Kountchev has been working with Prof. Leonid Yaroslavsky, a regular visitor of Tampere International Center for Signal Processing.

Prof. **Zoran Bojković** of Faculty of Transport and Traffic Enginering, University of Belgrade, Serbia, is another associate of Prof. Rao that was present. Thus, we asked him about his cooperation with Prof. Rao.

Prof. **Roumen Kountchev**
Technical University of Sofia
Bulgaria

Education
Engineer, M Sc., Electronics, Technical University-Sofia
Ph.D., Institute of Telecommunications, St. Petersburg, Russia. Thesis "Digital Methods for Structure Analysis of the Objects in the Image." Doctor of Science, Technical University of Sofia. Thesis "Inverse Pyramidal Image Decomposition: Methods and Algorithms".

Please tell us about your research in the field of spectral methods and especially about the Complex Hadamard transform?

Spectral methods are wide basis for the creation of a variety of efficient algorithms for digital image processing, analysis and compression. My initial investigations in the area of the spectral image transforms were aimed at the analysis of the properties and the applications of the Discrete Hadamard and the Karhunen-Loeve transforms, first presented in the monographs of N. Ahmed and K. Rao, H. Andrews, P. Wintz. My first publications in this area were in 1974-75 with the co-authorship of my wife, Dr. Roumiana Kountcheva. Together with her we patented "Method and apparatus for 2D Hadamard transform of TV signals". I defended my PhD thesis "Methods and analysis of objects structure in the image" in 1975, and the Electrotechnical Institute of

Telecommunications in Leningrad (Russia). My tutor there was Prof. Jakonia. The basic part of my dissertation was devoted to the development of methods and algorithms for objects classification based on spectral properties, obtained using the 2D Hadamard transform. The reviewers of my PhD thesis were Prof. Goldenberg from same Institute and Prof. Yaroslavsky from the Russian Academy of Sciences in Moscow.

My collaboration with Prof. Yaroslavsky continued later in the time of his visits to Sofia (Bulgaria). In 1988 were published (in Russian) the Proceedings of the Russian Academy of Sciences (Institute of Information Transfer): "Adaptive methods for image processing" by publishing house "Nauka", Moscow, with editors V. Siforov and L. Yaroslavsky. In the proceedings was included my chapter "Adaptive methods for TV images processing based on local differences". In 1995 was published my paper, in which I presented a generalized spectrum model for image representation based on the Inverse Pyramid decomposition (IPD). In this model could be built in various truncated orthogonal transforms: Fourier, Hadamard, DCT, Hartley, KLT, etc. The investigation of the spectral image models was the object of my second dissertation (for Doctor of Sciences), which I successfully defended in 2002 at the Technical University of Sofia (Bulgaria). In the next years were developed

1. The Reduced Spectral Pyramid (RSP) of non-overcomplete kind, based on the Walsh-Hadamard transform.
2. The Branched RSP, based on the preliminary histogram modification for a selected part of the initial components of the multi-channel images.
3. Non-linear IPD, in which could be built in various non-linear modified transforms: Mellin-Fourier, Kernel PCA, etc.
4. Non-linear IPD, based on the use of 3-layer neural networks with error

back-propagation and small number of nodes in the hidden layer.

On the basis of IPD implemented with linear transforms, were developed significant number of algorithms for efficient compression of natural, multispectral, multi-view and medical images, scanned documents, fingerprints, video sequences from surveillance TV cameras, etc. The nonlinear IPD were used for the development of new kinds of object descriptors, invariant to RST transforms and luminance changes. For the enhancement of the objects search in large image databases was developed the approach based on the multi-layer difference between images, represented by IPD models. Together with my wife and PhD students from the Laboratory for Image Processing at the Technical University of Sofia (Bulgaria) was developed the "Adaptive Color Karhunen-Loeve" Transform (ACKLT), which combined with the IPD was used for efficient still image compression and for objects color segmentation also (human skin, etc.). The combination of the ACKLT-based segmentation and classifiers, based on the Support Vector Machine (SVM), ensured high accuracy for human age recognition. Significant part of our investigations is related to the analysis of properties and applications of new class of orthogonal transforms, based on the Complex Arranged Hadamard matrices. It was determined that the use of the Complex Hadamard transform in the IPD permits to implement new kind of digital multi-layer phase watermarking of still images, distinguished for its high watermark capacity, practical invisibility (watermark transparency) and resistance against fraud attempts.

In conclusion, I want to note that except for the large number of publications (papers and monographic issues) related to the spectral methods, I have also three related patents: on the IPD, for multilayer phase watermarking of multimedia signals and for lossless image data compression.


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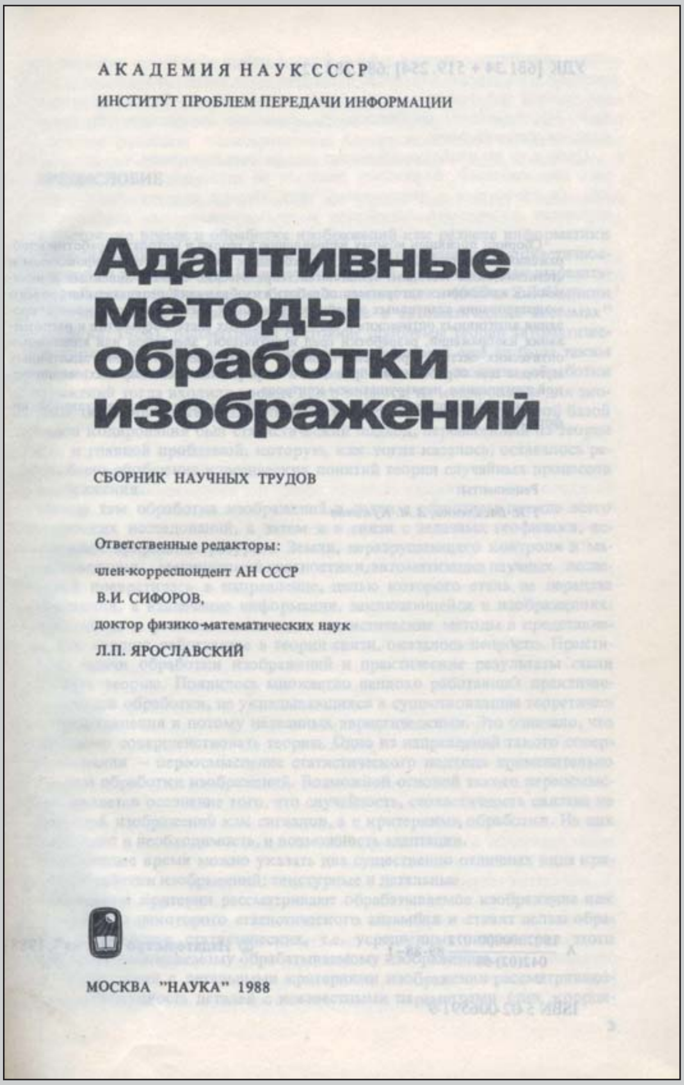
Адаптивные методы обработки изображений



АКАДЕМИЯ НАУК СССР

ИНСТИТУТ ПРОБЛЕМ ПЕРЕДАЧИ ИНФОРМАЦИИ

Адаптивные методы обработки изображений

СБОРНИК НАУЧНЫХ ТРУДОВ

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УДК 621.397

АДАПТИВНЫЕ МЕТОДЫ ОБРАБОТКИ ТЕЛЕВИЗИОННЫХ ИЗОБРАЖЕНИЙ НА ОСНОВЕ ЛОКАЛЬНЫХ РАЗНОСТЕЙ

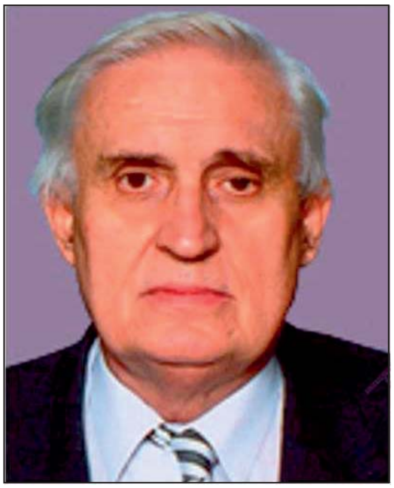
Р.К. КУНЧЕВ

ВВЕДЕНИЕ

Для построения эффективных методов фильтрации, коррекции и сжатия телевизионных изображений используют их определенные свойства. В соответствии с кусочно-линейной моделью изображений [1] они преимущественно характеризуются свойствами непрерывности и гладкости, которые нарушаются только на контурах. Это требует, чтобы алгоритмы обработки были согласованы с локальной структурой телевизионных изображений путем постоянной адаптации к ней. Возникает проблема выбора методов оценки локальных характеристик телевизионных изображений, обладающих достаточно высокой точностью и быстродействием, на основе которых можно разработать соответствующие алгоритмы обработки. Описанные в публикациях [1–4] адаптивные методы одномерной и двумерной (внутрикадровой) обработки телевизионных изображений имеют сравнительно низкую эффективность, а методы, изложенные в статьях [5, 6], используют трехмерную (междукадровую) обработку, для реализации которой требуются довольно сложные технические решения.

Предложенные в настоящей статье адаптивные методы обработки телевизионных изображений [7–9] подходят для реализации в реальном масштабе времени, имеют высокую эффективность при подавлении шумов, коррекции резкости и сокращении информационной избыточности телевизионных изображений. Эффективность этих методов обусловлена выбранным способом адаптации на основе определенных разностей элементов, находящихся в окрестности каждого элемента изображения.

Для оценки локальной структуры изображения применяются различные критерии, основанные на вычислении коэффициентов корреляции [10], дисперсии и интервала корреляции [11], усредненных градиентов в двух или больше направлениях по отношению к текущему элементу или поряд-



Zoran S. Bojković received his B.Sc., M.Sc., and Ph.D. degrees all in electrical engineering from the University of Belgrade in 1964, 1974, and 1978, respectively. After 4 years of work as a project telecommunication engineer at the Iskra Company, Kranj, Republic of Slovenia, he joined, the University of Belgrade. From 1987, he is full professor of Electrical Engineering. Also, until now he is a permanent visiting professor at the University of Texas at Arlington, TX, USA, EE Department, Multimedia Systems Lab.

Professor Bojković, what are your experiences with spectral methods? This is the topic here. When you started with spectral methods?

Yes, generally speaking I started with my master thesis and continued with my PhD thesis at the University of Belgrade. And I was very happy to have as an advisor Professor Georgije Lukatela. Perhaps you know him, of course, and he insisted, and he forced me to deal with digital image compressing, especially with digital image compression and rate-distortion theory. In that way, it was my master and I continue with PhD studies. Also I had a very good opportunity to meet Professor Robert Gray, from the University of Stanford, California. And after my PhD I spent some months with him in Stanford University, and it was a very good opportunity for me to enter very deep in the rate-distortion theory and also in the data compression signal. Then, after my thesis and during my master and PhD thesis, especially I deal with epsilon entropy. And as you know I hope, the founder of epsilon entropy was Professor Claude Shannon and also Kolmogorov, from Russia. After my thesis, I went to present the results, it was professor Lukatela who insisted upon, to present my results to Soviet Union Academia. And it was one conference in Georgia in Tbilisi, and then there I had opportunity to see Professor Kolmogorov which was for me something very, very great.

Unfortunately, I did not know Professor Shannon, but I know very well his work. And of course I was impressed. And then of course I continued my work in the domain of image processing. At the very beginning of 90s, I had also an opportunity to see in our bookstore in Belgrade, called Jugoslovenska knjiga (Yugoslavian book), one book from United States, which was imported from United States and it was a book Rao and Yip, the Discrete Cosine Transform. I was cited in this book on my work present at a a conference in France, and after that I decided to write to Professor Rao and to try to make some relations with him. I was invited to visit his laboratory at the University of Texas at Arlington. And from the first moment I was impressed. First of all, with Professor Rao, as a person, then as a Professor, after that with his family, especially his wife, who is present here, Madam Karuna. They accepted me very, very, very well and so after that I continue to go every time even when it was very difficult for us from Serbia to go to the United States. For example, in 1990, since it was the period of Milosević.

Slobodan Milošević was the President of Socialist Republic of Serbia and Republic of Serbia from 1989 until 1997 in three terms and the President of the Federal Republic of Yugoslavia from 1997 to 2000. The inexcusably wrong politics of Milošević and his followers and supporters and their incapability to learn from the past and understand the change in the international relationships caused many terrible problems to Serbian people. (Comment by R.S. Stanković)

I succeedto have a visa, and it was a very difficult to get it and I was very happy. I must say I was very happy when getting a visa for United States. Today it is not a problem. You can have a visa for one year, two years, five years, and so on. And step by stepwe start our work. I had some proposal for writing books, and all my proposals were accepted, with Professor Rao. So, we stared in1999 with one book edited by Prentice Hall, the other one was three years ago, also about introduction to multimedia communication, then we published with Wiley and now with CRC Pres and so on.

I should like to point out here that after the revolution in Serbia, I was accused [by some colleagues] that I was a collaborator with Milošević. And it was horrible for me.
And I must say, perhaps it is for the first time for Professor Rao and for his wife to hear that I was accused. They told me, during Milosević, you spent, uh, our money, you went to USA and so on and so on. Uh, it was not true, of course, and, uh, I must say that when I was accused [by some colleagues] , it was after the revolution, in 2000, it was a very difficult time for me. They told me you are a collaborator with Milošević, and in one moment, I caught them, and showed them the book Packet Video Communications over ATM Networks, and it is by Prentice Hall, and I told them, when you were sitting on bridges all the time, and prying the God, what would be, I went to the USA, to write books. When I show them this book, with Professor Rao, published by Prentice Hall, USA, they did not tell me anything and my rating became higher and higher. I must say that owing to Professor Rao and his labs in United States of America I succeed to improve my rating here. I was very happy because of it, but this is the fact.

Thank you for giving me the opportunity to speak about it.

Zoran S. Bojković, *Digital Signal Processing,*
Naučna knjiga, Belgrade, Serbia, 1989
(in Serbian).

Professor K.R. Rao and his students
Photo from the web page of Prof. Rao.

Professor K.R. Rao and his students
Photo from the web page of Prof. Rao.

Professor K.R. Rao and his students
Photo from the web page of Prof. Rao.

Professor K.R. Rao and his students
Photo from the web page of Prof. Rao.

Dr. Rao supervises his 100th graduate student Vidhya Vijaykumar.

Photo from the web page of Prof. Rao.

Gallery of Books authored, co-authored, and edited by Professor K.R. Rao

The gallery of books starts with reprints of the cover, title pages, and few pages of the Russian edition of the book N. Ahmed, K.R. Rao, *Orthogonal Transforms for Digital Signal Processing*, Springer, 1975.

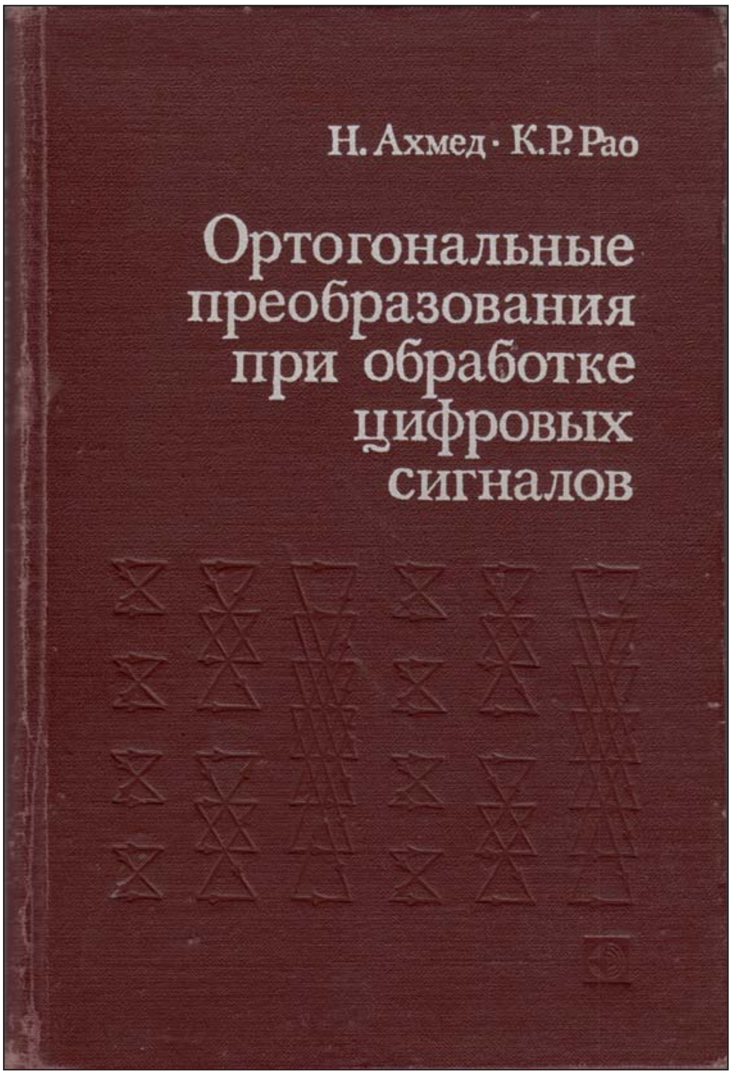
The translation in Russian of this book was published by the renowned the publishing house Svyazi, Moscow, in 1980. The book was translated from English into Russian by Prof. T.E. Krenkel, who was a visitor of TICSP several times. The translation into Russian was supervised by I.B. Fomneko, as it can be seen at page 62.

At page 63, the UDK classification of the Russian edition is shown.
The preface to the Russian edition, written by Prof. A.M. Trakhtman, is shown at pages 64 and 65, while the page 66 shows the translation of the preface to the English edition as written by the authors.

The Russian edition contains an additional list of references prepared by the experts in the former Soviet Union, that are reprinted at pages 66 to 69 as well as the first page of the index.

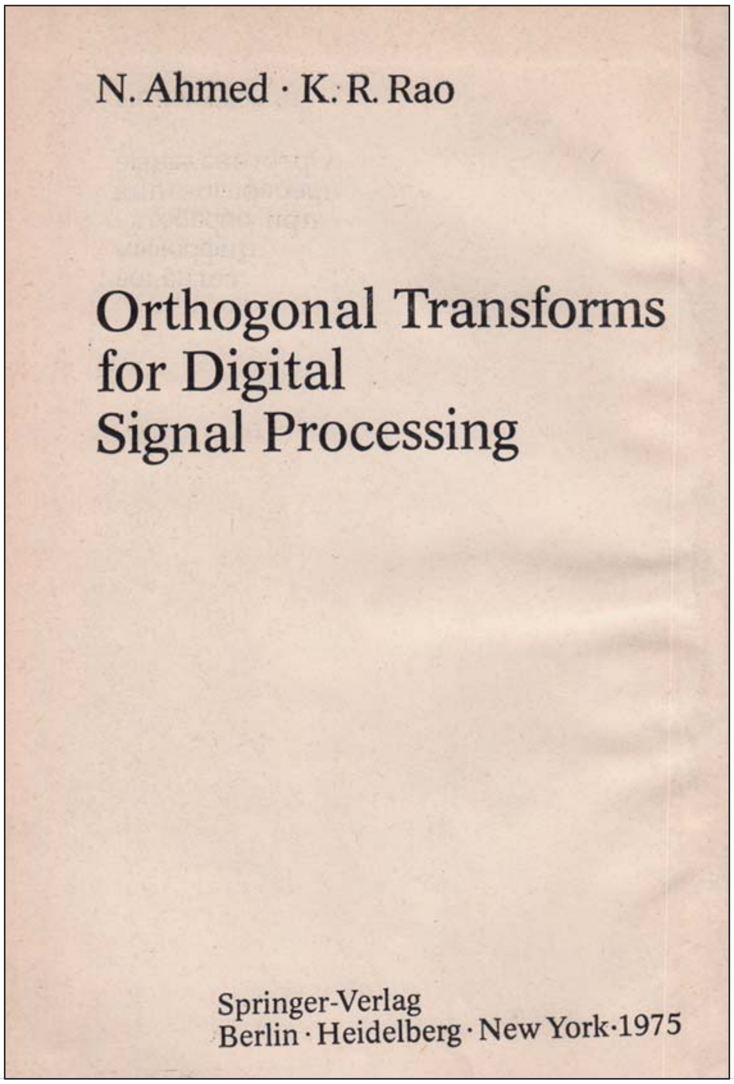
Page 70 shows the impressum, where it can be seen that the book was published in 5000 samples, and the price per sample was 1 ruble and 40 kopek.

The Gallery continues with covers of several books authored, co-authored, and edited by Prof. K.R. Rao.


Н. Ахмед · К.Р. Рао
Ортогональные преобразования при обработке цифровых сигналов

Н. Ахмед · К.Р. Рао

Ортогональные преобразования при обработке цифровых сигналов



N. Ahmed · K. R. Rao

Orthogonal Transforms
for Digital
Signal Processing

Springer-Verlag
Berlin · Heidelberg · New York · 1975

Н. Ахмед · К.Р. Рао

with best compliments

K R Rao (9, oct. 2009)

Ортогональные преобразования при обработке цифровых сигналов

Перевод с английского Т. Э. Кренкеля.
Под редакцией И. Б. Фоменко

МОСКВА ·СВЯЗЬ· 1980

ББК 32.88
А95
УДК 621.395.4

Ахмед Н., Рао К. Р.

А95 Ортогональные преобразования при обработке цифровых сигналов: Пер. с англ./Под ред. И. Б. Фоменко. — М.: Связь, 1980. — 248 с., ил.

В пер.: 1 р. 40 к.

Книга написана ведущими американскими специалистами в области перспективного направления ортогональных преобразований цифровых сигналов, все шире внедряемых в системах связи, телефонии, телевидении, радиолокации, телеметрии и т. д. Это своего рода пособие для знакомства, изучения, обоснованного выбора и инженерных применений различных ортогональных преобразований при обработке цифровых сигналов.

Книга предназначена для инженеров и техников, специализирующихся в области обработки сигналов.

А $\frac{30602-138}{045(01)-80}$ 9—80 2402000000

ББК 32.88
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ПРЕДИСЛОВИЕ К РУССКОМУ ИЗДАНИЮ

Авторы настоящей книги Н. Ахмед и К. Р. Рао одни из первых активно включились в 70-х гг. в разработку вопросов, связанных с применением функции Уолша при цифровой обработке сигналов. Однако в отличие от других исследователей они изучали возможность применения для цифровой обработки также и других систем ортогональных дискретных функций. О плодотворности работ авторов в этой области можно судить по тому, что за короткий период 1970—1974 гг. ими было опубликовано более 30 журнальных статей, посвященных функциям Уолша, различным обобщенным преобразованиям, преобразованиям BEFORE, дискретным косинусным преобразованиям, преобразованию Хаара, методам вычисления спектров в различных базисах, применению спектральных методов для решения отдельных задач и т. д. Эта активность продолжается до сих пор, поэтому Ахмед и Рао чрезвычайно популярны среди специалистов, занимающихся цифровой обработкой сигналов. Основные результаты своих исследований до 1975 г. они обобщили в данной книге, которая была задумана как учебное пособие для студентов, специализирующихся в области цифровой обработки сигналов. Эту задачу авторам удалось выполнить в полной мере — книга написана кратко и понятно, хорошо методически увязана, содержит много задач.

С момента выхода книги в свет прошел немалый срок для такой бурно развивающейся области, как цифровая обработка сигналов. В этот период развитие теории ортогональных преобразований стимулировалось, с одной стороны, новыми практическими задачами, которые удобно решать цифровыми методами, и, с другой стороны, — новыми техническими возможностями, которые открыл прогресс микроэлектроники, неуклонно развивающейся по пути увеличения степени интеграции элементных средств.

Следует отметить следующие важнейшие результаты, полученные за период с 1975 г. в теории ортогональных преобразований, которые, естественно, не отражены в данной книге. Значительно расширен ассортимент изученных базисных функций. Построена теория дискретных функций Виленкина—Крестенсона и обобщенных функций Хаара на конечных интервалах. Развита теория многомерных преобразований. Синтезированы базисные функции, позволяющие решить отдельные частные задачи оптимизации, т. е. приводящие к спектру, близкому к классическому, но более экономные в вычислениях, дающие наиболее сжатое описание сигналов, минимизирующие ошибку вычислений, минимизирующие объем аппаратуры, обеспечивающие уникальные характеристики аппаратуры и т. д.

5

Более глубоко раскрыта природа быстрого преобразования Фурье и быстрой свертки — центральных операций почти всех систем цифровой обработки сигналов. На основе этих преобразований разработаны новые модификации алгоритмов быстрых преобразований и расширены возможности их технической реализации. Развиты теория и практическое применение теоретико-числовых преобразований, позволяющих одновременно уменьшить и объем вычислений, и их ошибки. Успешно развиваются спектральные методы в теории кодирования, теории графов, теории функций многозначной логики.

В результате всех этих достижений, в которые немалый вклад внесли советские ученые, теория дискретных сигналов оформилась сейчас как новое и исключительно перспективное научное направление, в котором сигнал задается на множестве точек, представляющем собой группу (чаще всего — абелеву, с групповой операцией в виде специфического сдвига), значения сигнала задаются как элементы кольца или поля чисел (чаще всего — поля Галуа), а базисные функции задаются как характеры группы, на которой определены сигналы. Одним из важных проявлений такого обобщения понятия цифрового сигнала являются уже упомянутые теоретико-числовые преобразования, в которых структуры группы точек определения сигнала и числового поля возможных значений сигнала оказываются взаимно связанными. Этот наиболее общий подход к ортогональным цифровым преобразованиям уже дал многообещающие результаты и в будущем несомненно даст их еще больше.

Краткий перечень последних достижений приведен с целью подчеркнуть, что успешное овладение последними достижениями возможно только после некоторой предварительной подготовки, важной частью которой может служить изучение книги Ахмеда и Рао. Кроме того, книга Ахмеда и Рао сохранила свое значение для решения ряда частных задач, для которых она может оказаться вполне достаточной.

Благодаря строгой математической трактовке вопросов, совершенному матричному аппарату исследований и «вечной» актуальности рассмотренных практических аспектов (винеровская фильтрация, сжатие данных, распознавание образов) эта книга и сейчас может считаться ценным первоначальным учебным пособием для исследователей, инженеров и студентов.

О достижениях последних лет в области ортогональных преобразований при цифровой обработке сигналов дает представление список дополнительной литературы, приводимой переводчиком.

Доктор технических наук, профессор
А. М. Трахтман

ПРЕДИСЛОВИЕ АВТОРОВ

Эта книга предназначается для всех желающих получить сведения о практическом применении ортогональных преобразований в области цифровой обработки сигналов. Авторы надеются, что настоящая книга будет содействовать работе в этом направлении специалистов, занимающихся подобными вопросами в различных областях науки и техники.

Книга состоит из десяти глав. Первые семь глав посвящены изучению основ, обоснованию и описанию различных ортогональных преобразований, для понимания которых у читателя предполагаются знания по дискретному преобразованию Фурье (например, из курса дифференциальных уравнений) и матричной алгебре. Последние три главы посвящены некоторым специальным приложениям ортогональных преобразований в цифровой обработке сигналов. Для их понимания требуется знание основ дискретной теории вероятностей и элементов общей теории связи.

Бóльшая часть настоящей книги использовалась при чтении авторами курса лекций студентам старших курсов факультетов электротехники в течение последних пяти лет в университетах штатов Канзас и Техас. Первые семь глав соответствуют курсу лекций, рассчитанному на один семестр. Последние три главы могут быть положены в основу специального курса лекций, ориентированного на приложения ортогональных преобразований в цифровой обработке сигналов.

Так как настоящая книга возникла в результате чтения лекций, авторы выражают глубокую благодарность студентам старших курсов, прослушавшим лекции и в особенности Т. Натарайяну. Авторы благодарят сотрудников кафедры электротехники Канзаского университета и университета штата Техас за поддержку и заведующего кафедрой электротехники Канзаского университета доктора В. В. Копсела. Авторы считают своим долгом поблагодарить декана инженерного факультета доктора А. Е. Салиса, зам. декана доктора Р. Л. Такера и заведующего кафедрой электротехники доктора Ф. Л. Кэша университета штата Техас за финансовую, техническую и моральную помощь, оказанную авторам во время написания книги. Авторы благодарят также Дороти Бриджес, Анну Вулф, Маршу Пирс, Линду Душ, Евгению Джо, Еву Хупер, Дану Кэйс, Кэй Моррисон и Шэрон Малден за труд, связанный с печатью различных глав рукописи.

Наконец, авторы благодарят своих жен Эстер и Каруну, без моральной поддержки, терпения и понимания которых настоящая книга не могла бы быть написана.

Н. Ахмед, К. Р. Рао

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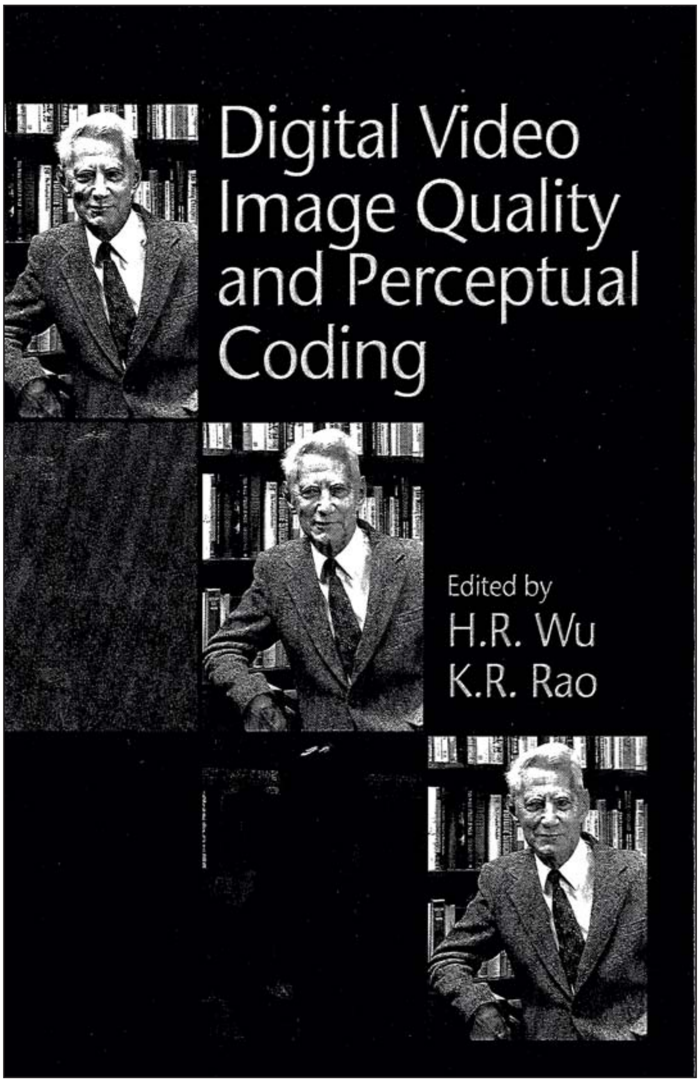
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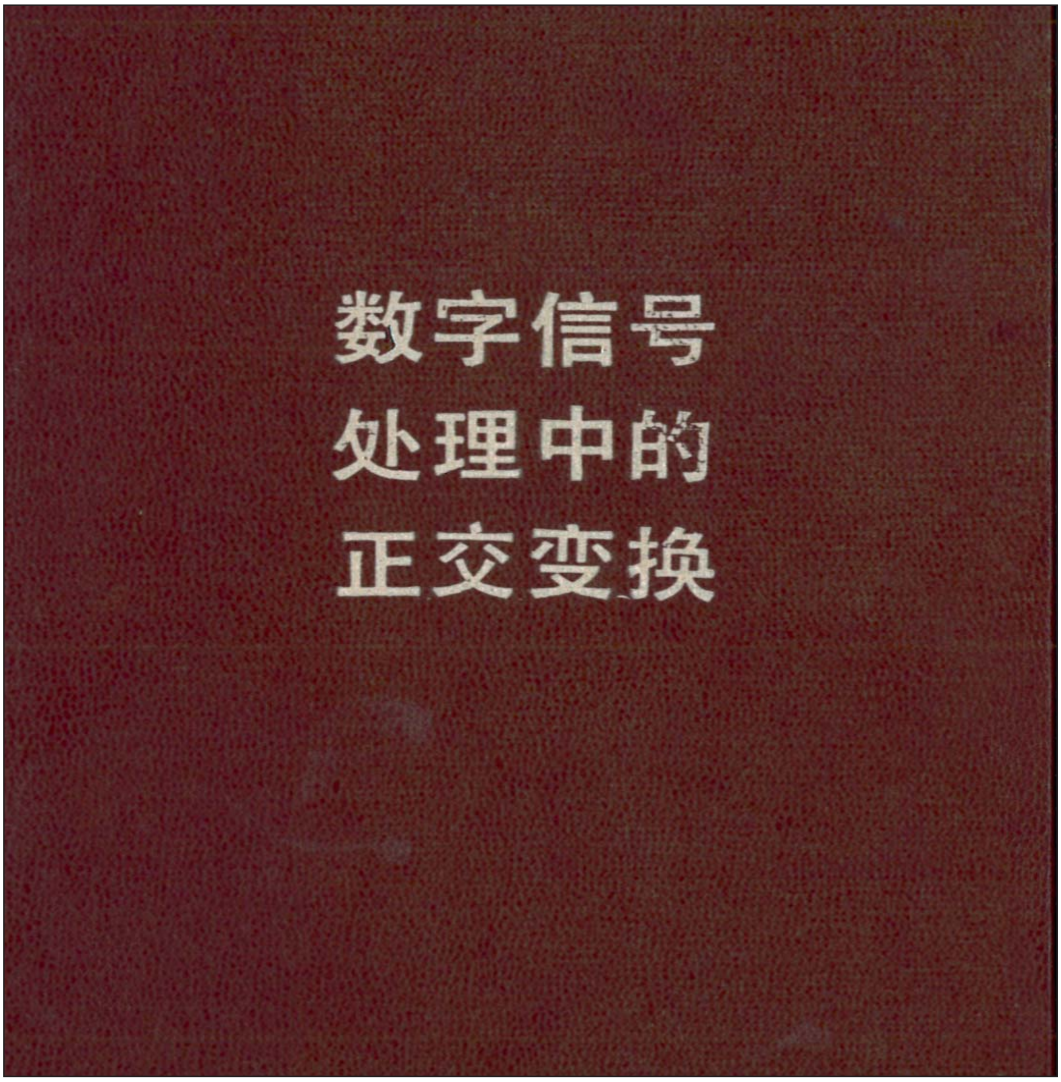
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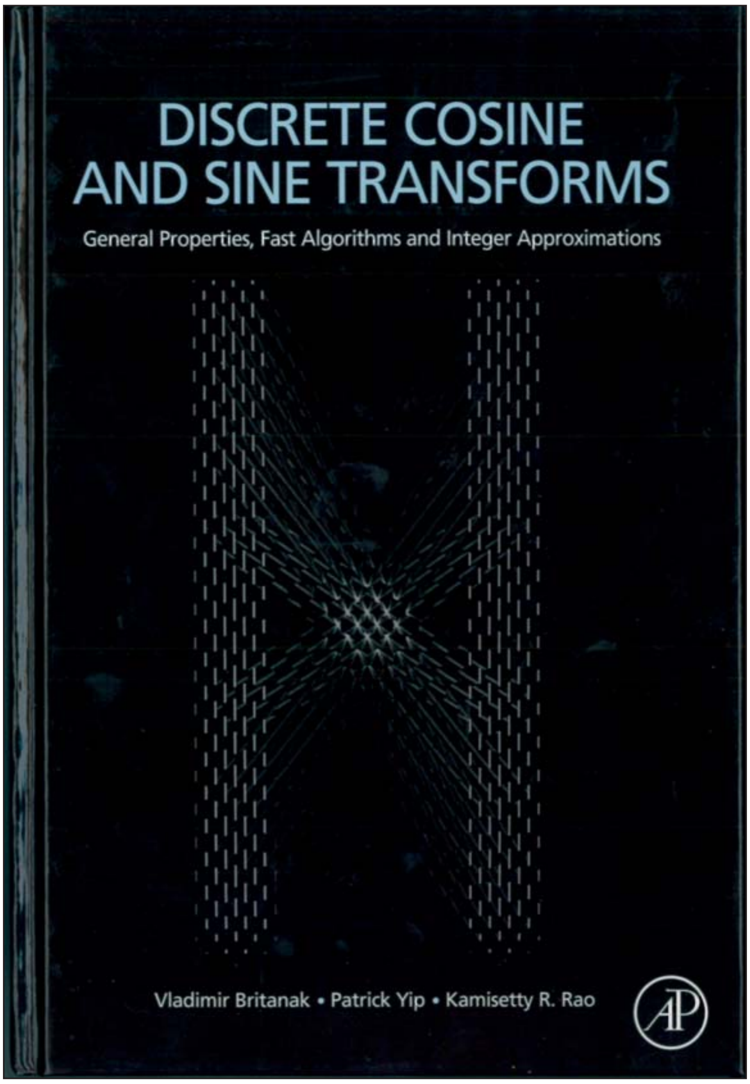
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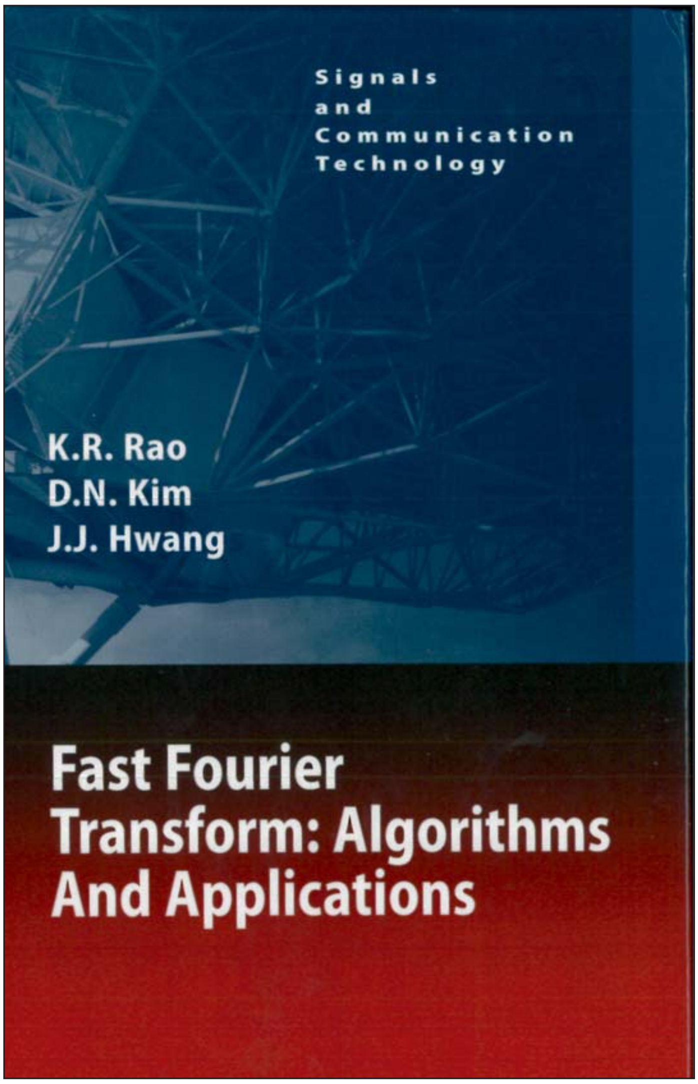
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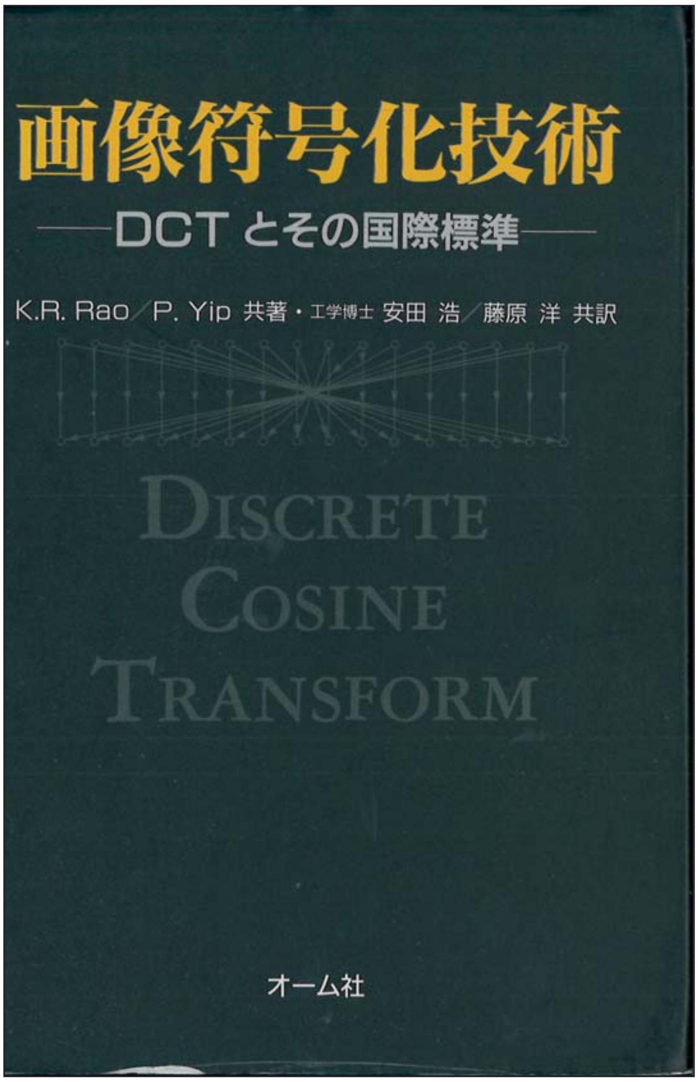
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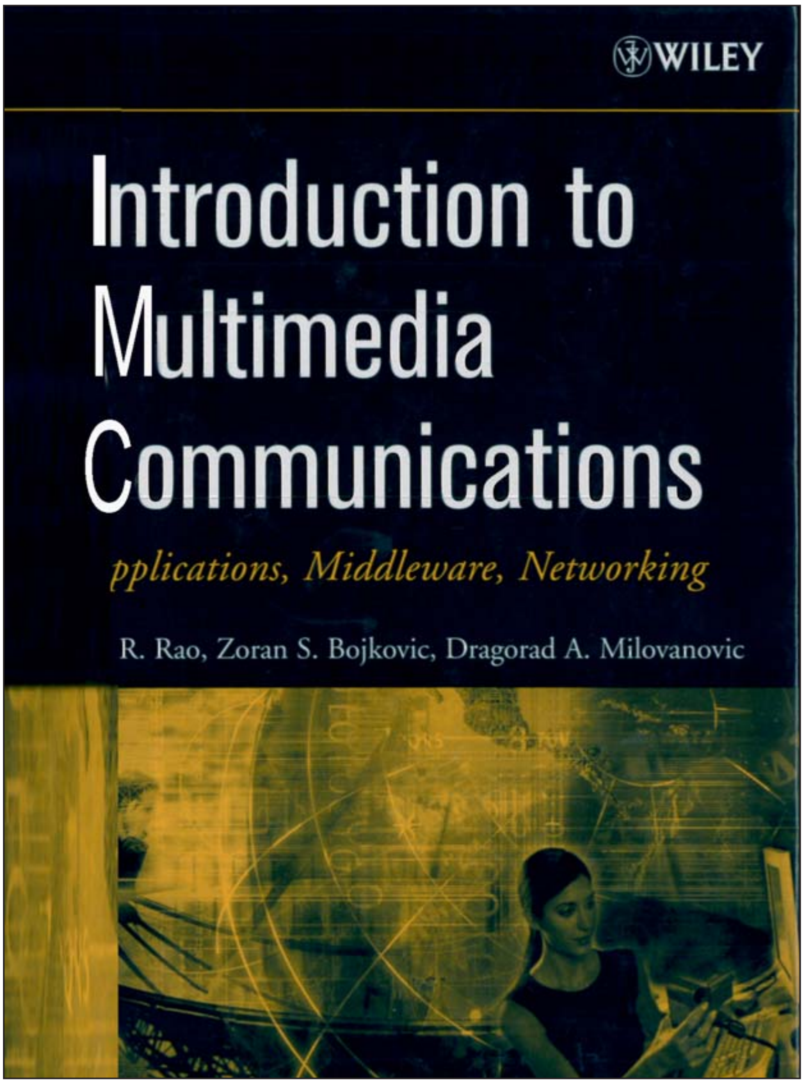
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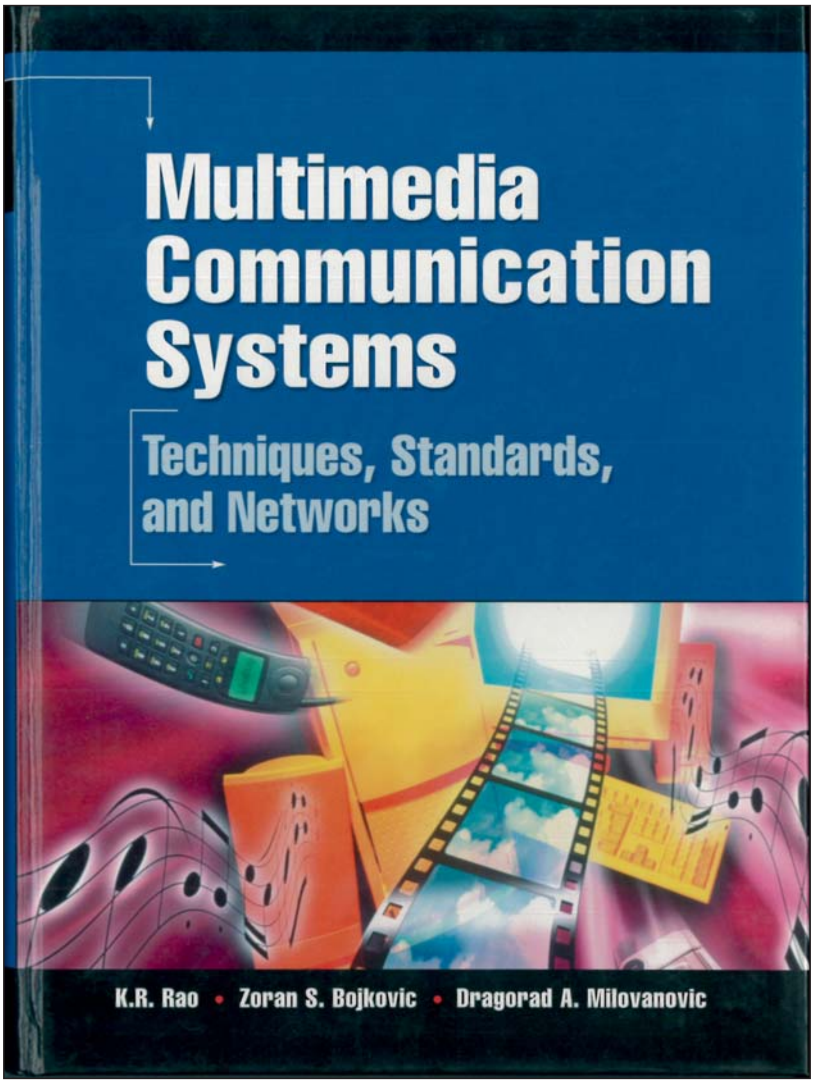
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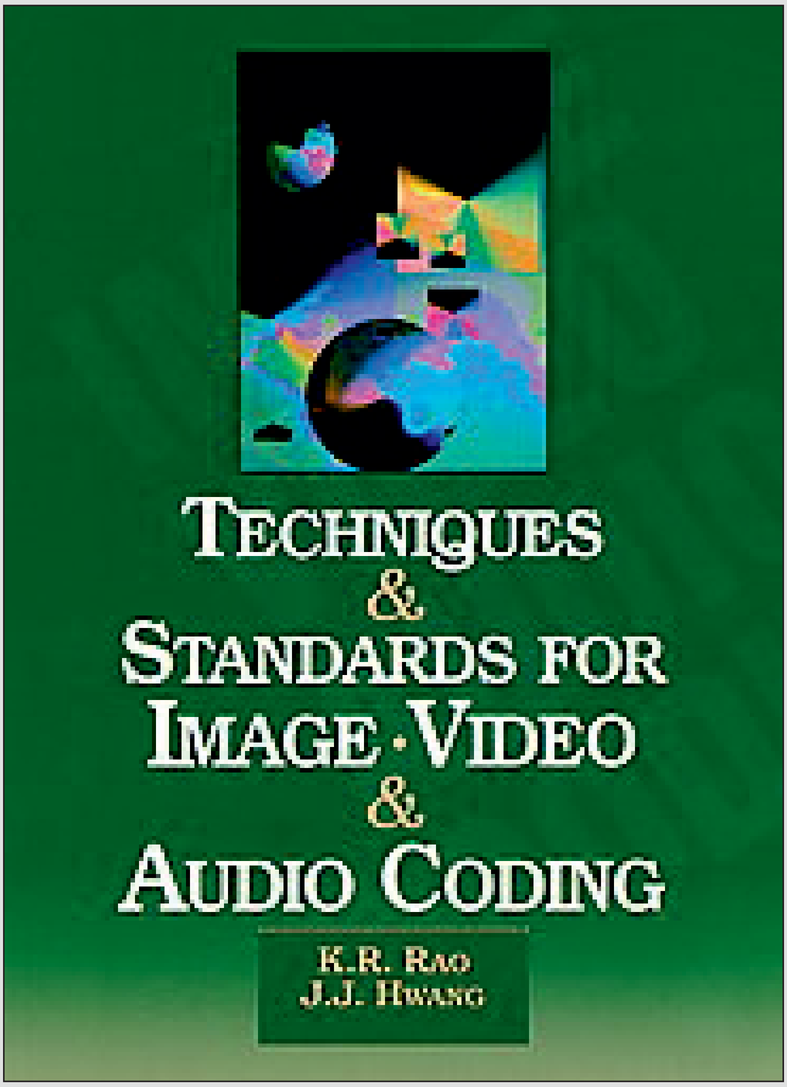
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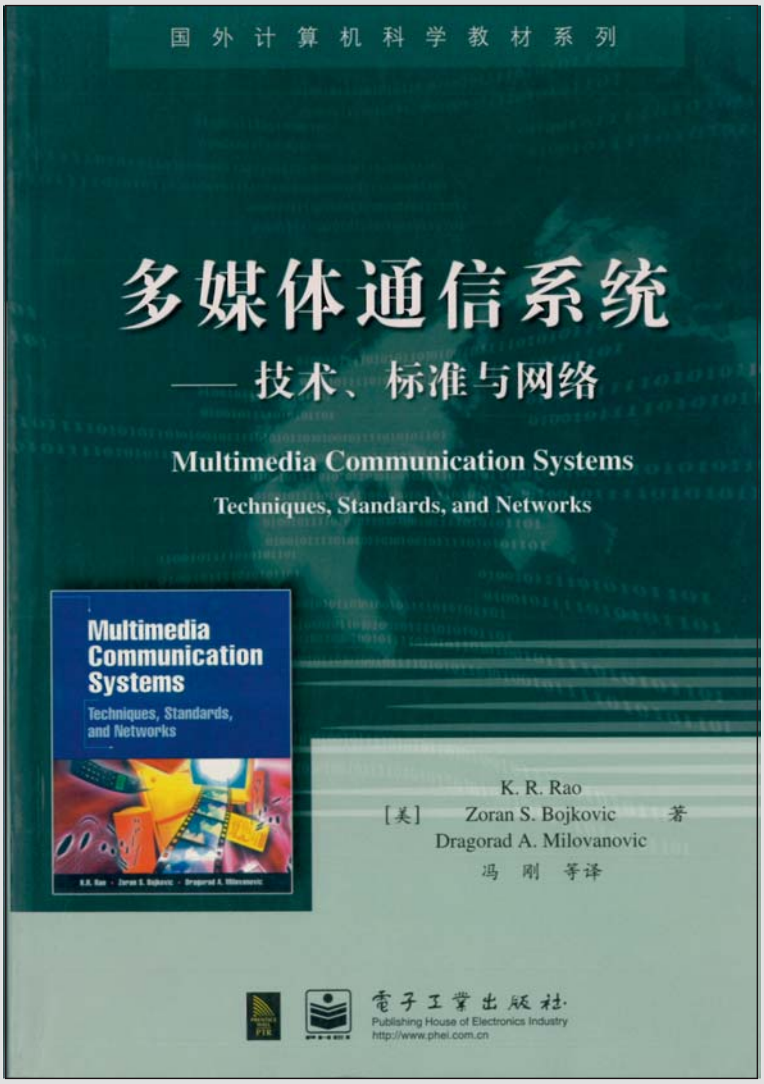
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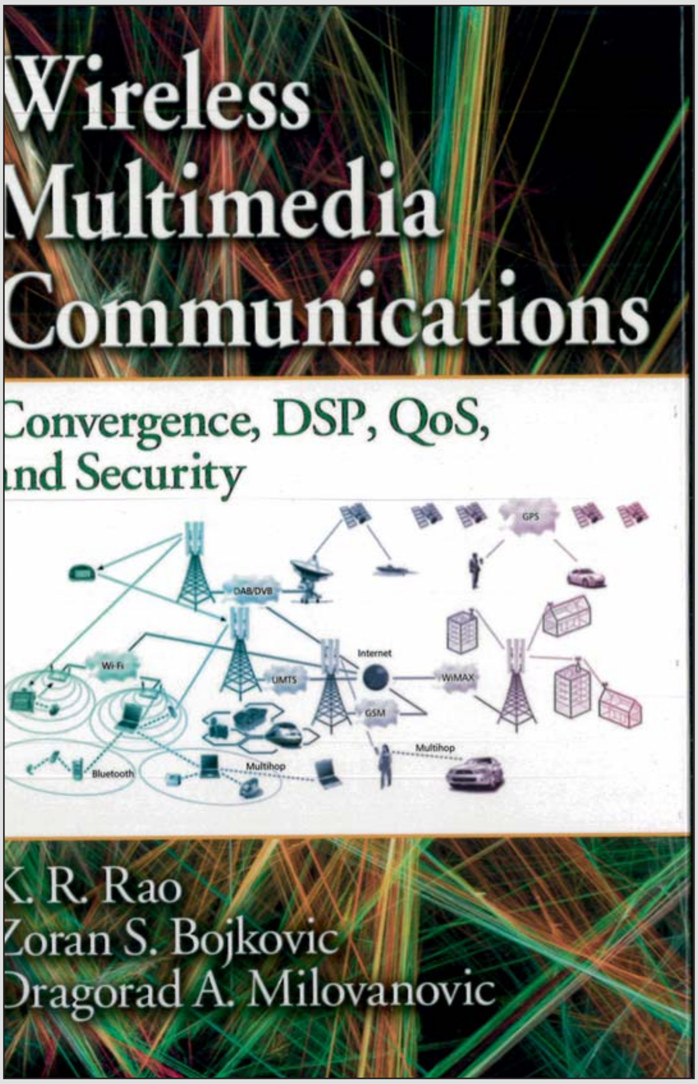
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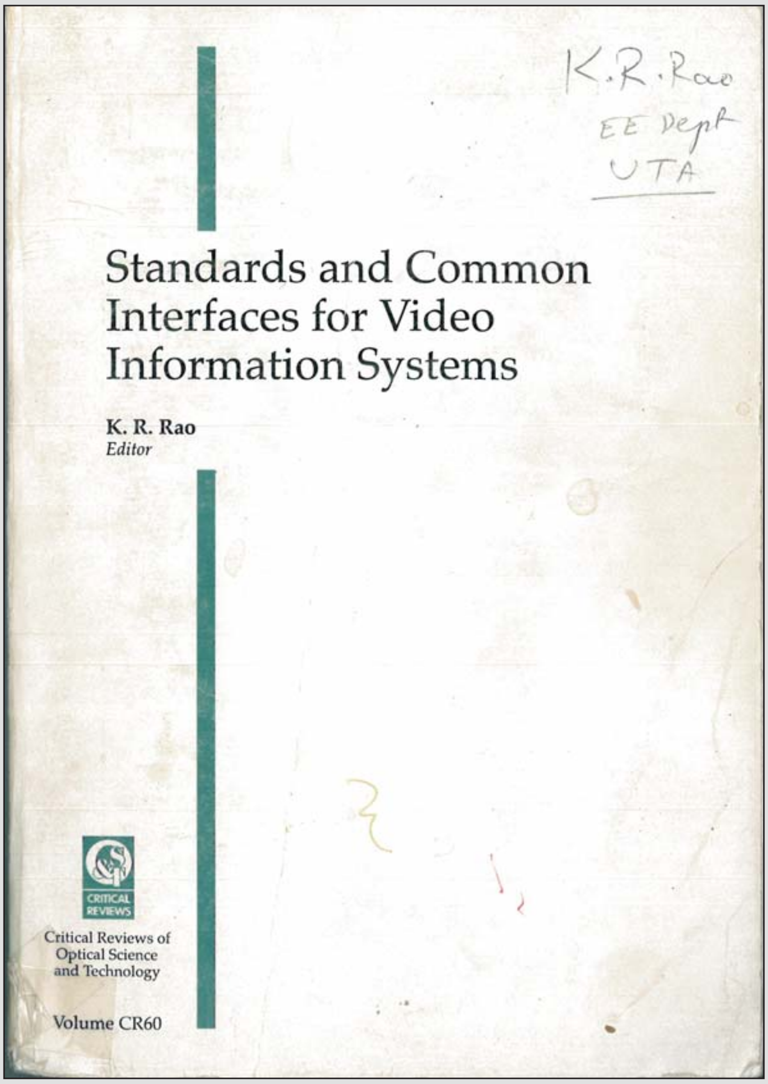
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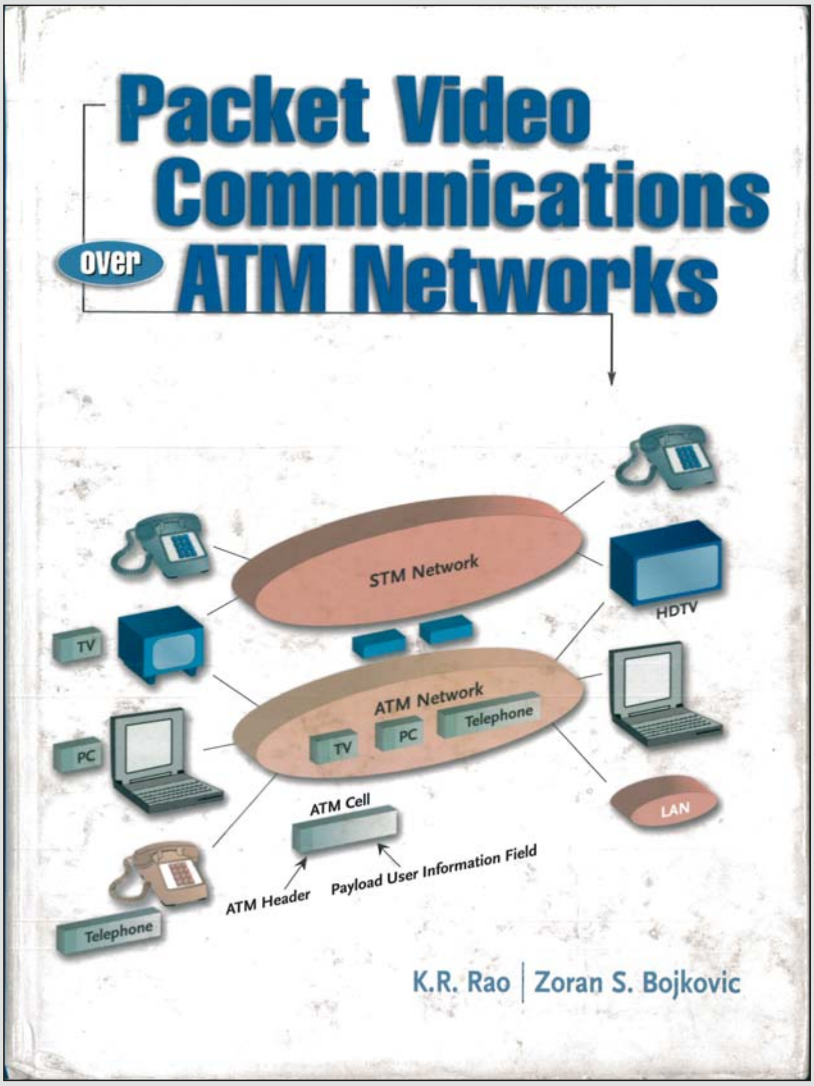
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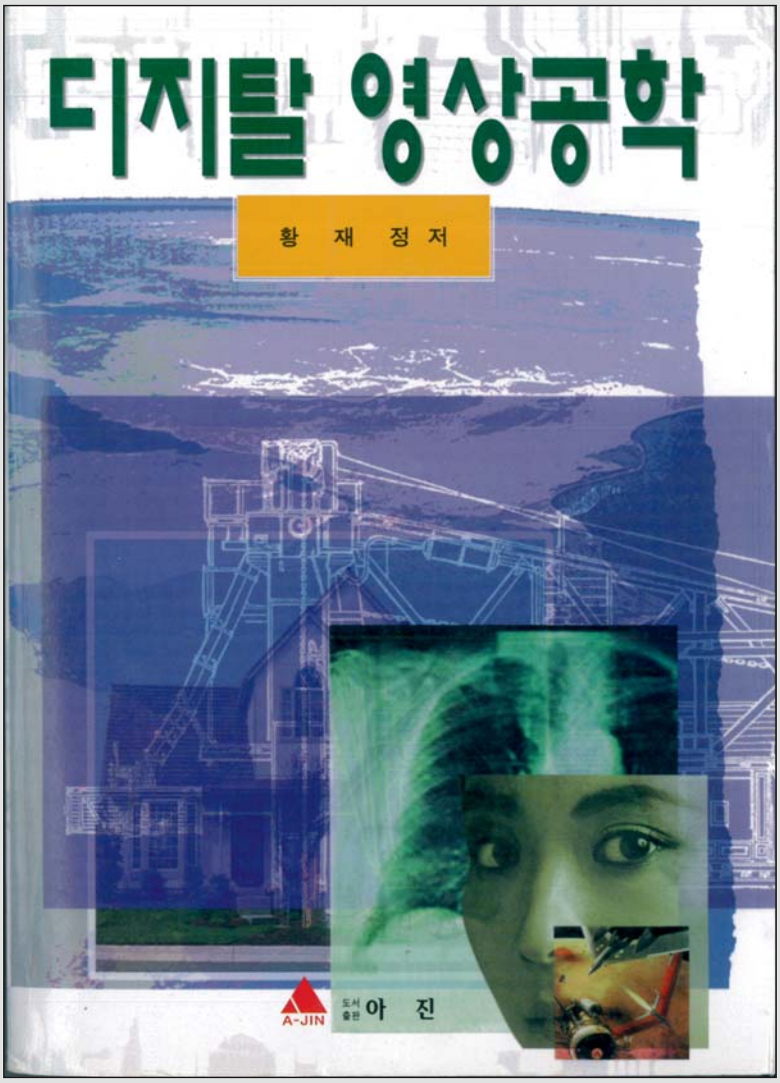
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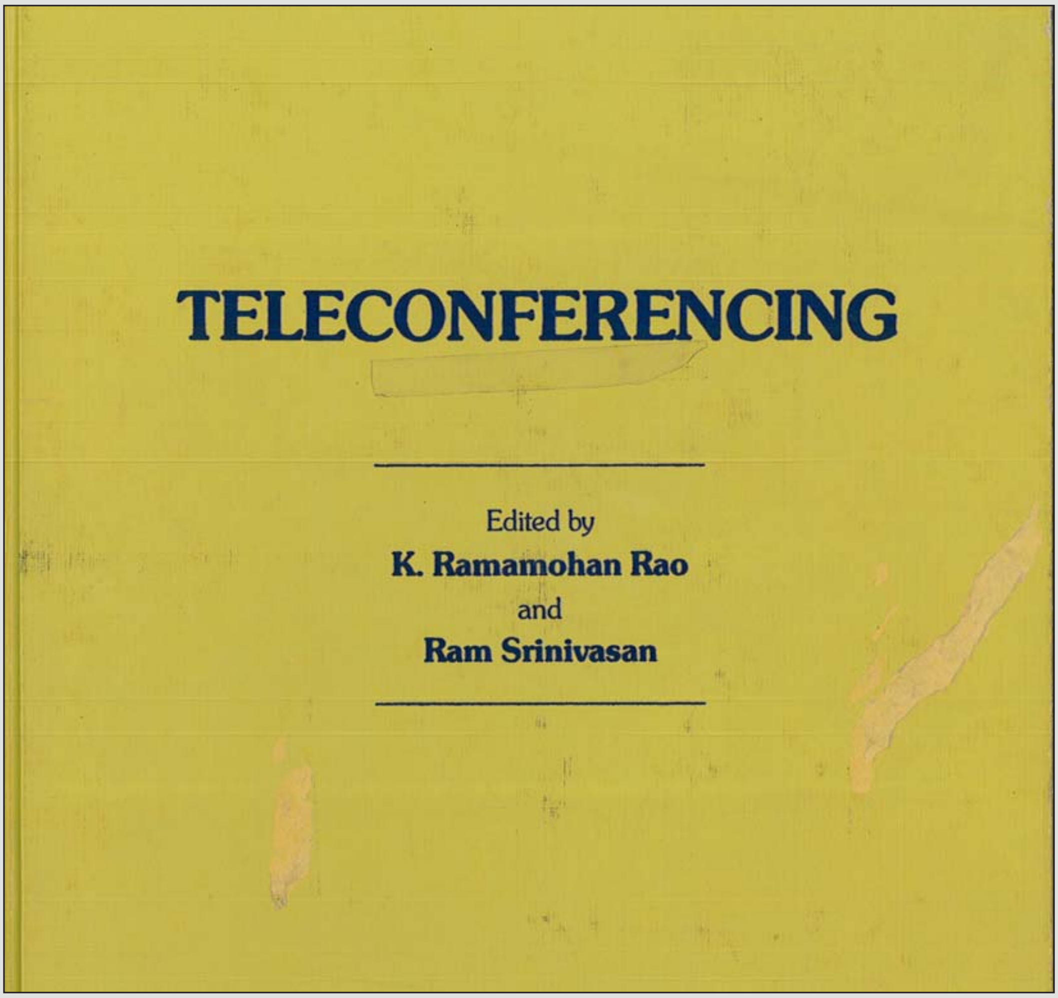
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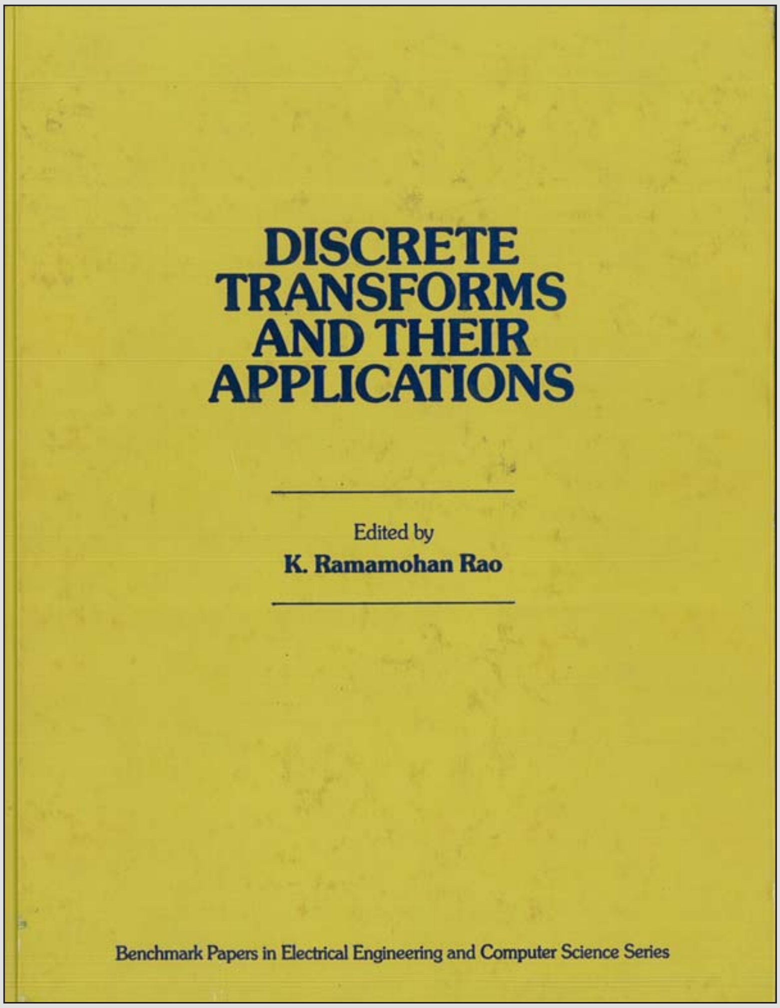
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