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Properties of the Single Conductor

New Fundamental Relations

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***Synopsis.**—The properties of a unit length of single, straight conductor, far removed from all other circuits, are investigated to endeavor to find whether such a unit is a basic, fundamental one, on which deductions and a method of mathematical treatment could be based, as was advocated by Ampere, to supplement (not to replace) the Maxwell system based on a complete circuit, and to test the correctness of some of the postulates now in common use, based on the latter system. By several new, simple, and direct proofs based only on a few well-established and accepted relations, chiefly the internal stresses in a conductor, but excluding infinities, self-inductances, induction, and any postulates, a constant is deduced for the energy stored by a current in such a unit length, which seems to be one of the most fundamental, basic constants in electrodynamics, from which many useful deductions can be made, some of which are given. This energy of the flowing current corresponds to the* $m\,v^2/2$ *energy of moving masses.*

Some of the results differ from those which have been in use; explanations are offered of the cause of these differences, and it is shown how the results may be brought into agreement, involving some changes in our previous conceptions. It is shown that with flux lines a distinction ought to be made which is analogous to that which distinguishes the wattless ampere from the one in phase, or between a true resistance and an impedance; it is shown why what might be called wattless flux, ought to be recognized. It is shown that self-inductance is used in two senses which may sometimes lead to different results, and that a distinction should therefore be made between them somewhat analogous to that between resistance and impedance or reactance. This it is believed would clear up the ambiguity now existing in that term. It is believed that some of these results could not have been obtained from the Maxwell complete circuit system, which rather leads one away from them.

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INTRODUCTORY

IN science, as in engineering and mathematics, it is always desirable to have two independent methods of getting a result; they not only afford a very desirable check on the result and on each other, but each one often has advantages over the other in the different fields for which it is best adapted. For the analytical treatment of electric currents and circuits, the "complete circuit" system of Maxwell has been taught universally and exclusively and for so many years that the strong though wrong belief has arisen among many that it is the only possible one, and that every case must be treated from that and only from that standpoint. In some cases this has not only misled us but has sometimes even led us away from useful facts and relations.

There is a second and older system, the one advocated by that great mathematical physicist Ampere, which is based on the single conductor, and which has many advantages over the other in specific fields for which it is better adapted. Neither system should be used to completely replace the other; they should be used to supplement each other, each in its own field. An electron may start from rest, move to another point and come to rest again there, as in a bolt of lightning; while moving, it is a current and generates a magnetic field in which energy is stored, quite analogous to the $m\,v^2/2$ energy stored in a moving body. The "complete-circuit" system is a misfit in such a case and involves complications which are burdensome and confusing to the student, while the single-conductor system applies directly.

On the other hand the complete-circuit system lends itself well to integrations around a completed path. It has served its purpose well and is very useful, and in most of the usual circuits it is quite reliable. But in a search for the true, basic, fundamentals, there is no such a thing as a fundamental or unit or limiting circuit; the nearest approach to one is a circular circuit in the form of a tore or torus whose centerline radius is equal to the radius of the wire, like a doughnut with an infinitely small hole through it. This is the shortest circuit which can be made with a wire of given diameter, hence is at least a limit, but for various reasons this is not satisfactory as an ultimate fundamental.

The writer has therefore concluded that the only real fundamental is a unit length of a long straight, single, conductor, far removed from all others, as this embodies absolute ultimata; it is the form which is most free from all external influences, and besides the current, it involves the least number of variables, in fact only one, the radius of the wire, and even that one falls out in some cases, as will be shown below, for relatively to the infinite free space around it for the magnetic flux, the small finite radius of the wire becomes negligible.

The purpose of the present paper is to examine analytically the properties of such a unit length of this fundamental conductor, without involving infinities, the limitations of complete circuit mathematics, self-inductances, induction, postulates, or any but well established relations, and to deduce from these properties any useful relations and constants that may exist.

To do so by considering a straight line as a special case of a circle of infinite radius, is unsatisfactory, as it introduces that dangerous quantity, infinity, with its serious pitfalls into which many otherwise able men

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have fallen[2]. In a French book by Fleury dealing with infinity he says: "Infinity has no other property than that of being impossible. Any calculation based upon absolute infinity, or upon any function whatever of absolute infinity, is itself absurd." The writer agrees with him in this, in part. When the results of two methods fail to agree, "look for the infinity" he says, and you will probably find the error.

When absolute infinity occurs, as it does, in mathematical deductions, there are two ways in which it can be safely applied in practise. The first one applies (as Fleury admits), when it is possible to assume that the quantity is simply so large (though finite) that others which depend on it may safely be taken as the same as they would be if it were infinitely large; this is often the case. For instance, in starting a current in circuits containing some small inductance, a few seconds is generally quite safely taken to be the equivalent of the eternity required theoretically for the current to reach its final value even in very accurate tests; the currents in even sensitive tests made in New York do not appreciably affect simultaneous researches made in Philadelphia, the 90 mile distance being practically infinite. Our atmosphere should extend theoretically to infinity, according to the laws of gases, yet beyond the relatively short distance of 50-60 miles there is not enough left to consider; the moon, sun, planets and stars encounter no friction in our atmosphere, even though it should extend theoretically to infinity.

The second one applies when some *property* of an infinite quantity can be found which is finite and therefore can be practically applied; thus the 760-mm. pressure of our atmosphere is finite, though according to the theory of gases, the atmosphere must extend to infinity; or similarly, the radial magnetic pressure on the surface of a single straight conductor, by the flux surrounding it, is known to be finite, although the flux, like the atmosphere, should extend theoretically to infinity. It will be shown later that under certain conditions the flux energy is also finite, though the flux is infinite.

One of the pitfalls in dealing with infinities obtained mathematically is that sometimes the real result is that the quantity is merely indeterminate and not infinite; this may arise when a factor has been suppressed or dropped because it is unity, but it is physically still present.

By treating the circle as a special case of a straight line, instead of the reverse, infinities and their pitfalls are avoided, which is another reason for preferring to start with the straight line as the fundamental.

The Single Conductor

In dealing with the single straight conductor as the fundamental, the real underlying condition (besides the straightness of a reasonable length) is not that it must be infinitely long, but merely that the unit length is assumed be so far removed from all other currents, or magnets, that they will have no appreciable effect on it; such currents include its own return circuit. Hence in a complete circuit in the form of a large square the sides may be made so large that a unit length l, Fig. 1, in the middle of a side, is no longer affected by the return current in the opposite side L.

The only three effects which neighboring currents can have on the unit length l, are attraction, repulsion and induction, which decrease as the square of the distance, and although they may also increase as the length L increases, yet the action of both combined is always a *decreasing* function, hence a distance may always be reached at which the effects are small enough to be neglected, as is always done when two or more independent tests with complete circuits are made at different places at the same time, even though with great accuracy.

Hence such a square can always be conceived in which the field at its center may safely be taken as

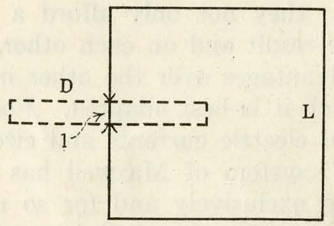

Fig. 1

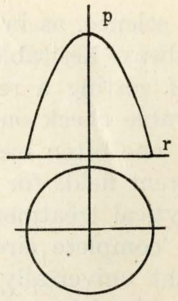

Fig. 2

zero, and therefore a unit length l in the middle of a side is practically as free from the effects of neighboring currents, as though it were a single conductor, and the flux and flux energy inherent with it are practically those in the disk-shaped space D between two parallel planes one unit apart, just as they are in a single conductor. Such a unit length l may therefore be taken to represent a unit length of a single conductor to any degree of accuracy desired by merely making the square large enough. Such a square circuit is therefore a connecting link between a unit length of a single, straight, conductor and a complete circuit; and the condition of infinite length, so often used in order to degenerate the single conductor to the realm of impossibilities, is no longer justifiable; some of the pitfalls of infinity are thereby avoided.

Maxwell himself determined (Art. 478) the now classic law for the flux density around a single conductor, with such a closed circuit; Northrup's determination of the internal pressures in a single conductor produced by the flux, is not based on the condition of infinite length, but as he says, merely on freedom from the effects of neighboring currents, and the accuracy of his expression was checked experimentally in a

2. This was well brought out in a discussion by Dr. C. O. Mailloux, Trans. A. I. E. E., Vol. 42, 1923, p. 328, Col. 2, par. 4, to p. 329, Col. 2, par. 2.

closed circuit. Ampere's able analytical researches with the single conductor were no doubt based on the same conditions; so is the generally accepted $i^2/4$ energy of the flux in the interior of a conductor. Many other like cases could be cited, showing that the infinite length is not a necessary condition. The same condition of freedom from all external influences is assumed in all accurate electrical tests, hence it is not an unreasonable or impossible condition.

Energy

Energy is the one physical quantity which is common to all physical phenomena and processes, and its absolute unit, the erg, applies to them all; hence it is the best and in fact the only connecting link between all the different groups of physical phenomena. Forces may be electromotive, magnetomotive, or ponderomotive (tending to move masses), each of which is physically different from the other, and their units are different and are not directly convertible. Moreover some forces are vector quantities, have components and resultants, can be generated or annulled and follow different laws, while energy is always a constant quantity no matter what form it may be in and its laws are common to all its forms. Energy is therefore the best physical quantity to use as a fundamental and the best connecting link between different physical phenomena.

Zero Resistance

In searching for fundamentals the number of factors or variables should of course be the least possible. At absolute zero of temperature the resistance of pure metals is eliminated, as it is zero; a current once started, as by induction, will therefore continue (theoretically) to flow forever, just as a body set into motion will continue to move when it meets with no resistance. Zero resistance will be assumed in the present discussion.

In the electric case this stored energy, which was stored while the current was started, is represented entirely by what seem to be stresses or strains of the ambient ether, which are called magnetism, hence it is magnetic energy and seems to be very closely analogous to the mechanical stresses or strains in compressed or stretched elastic rubber, and is static or potential in its nature. This stored energy of a current is closely analogous to the $m\ v^2/2$ energy stored in a moving body. When any of this stored energy, at zero resistance, is released, as by doing external work, the current also decreases, as shown in a paper by the writer,[3] and when it has all been released the current will become zero.

Such a condition of zero resistance simplifies greatly the considerations and conception of some electrical phenomena involving flux and energy, as there is then no external source of energy to mask the conditions when some of the stored energy has been released, as is the case when a "constant current" is specified in stating the laws, as it usually is, in which case the released energy is at once restored again, and there is also a continuous flow of energy. At zero resistance and with no connected source, the variables are reduced to their minimum, hence they are the best conditions for studying the fundamentals. The permeability is, of course, assumed to be unity, in all such fundamentals.

Unit Relations

The very basis of the absolute or c.g.s. system is to make as many as possible of the numerical relations between different fundamental physical quantities, unit relations, that is, the coefficient is unity, in which, however, the factor 2 often resulting from integrations, and π which cannot be avoided, must be included. If therefore, in the search for new fundamentals and relations between them, the latter turn out to be unit relations, the presumption is strong that they are correct, though it is not necessarily an absolute proof. A number of the relations deduced below will be seen to be unit relations.

The Fundamental Constant

One of the most basic fundamental quantities in electrodynamics therefore seems to be the energy represented by, or inherent with, or stored in, a unit length of single, straight conductor by a given steady current, under the most basic conditions; if this quantity leads to unit relations its fundamental character is confirmed.

Determination of this Constant. The problem which the writer endeavored to solve therefore was to determine this basic fundamental constant and to do so by a method which does not involve any physical quantities other than energy, mechanical forces, and currents, hence excluding completely that ambiguous quantity called self-inductance, and the theory of linkages (perhaps still incompletely proved) in the case of self-produced flux, both of which are so often relied upon absolutely in such energy calculations; nor does it involve any infinities or "postulates"[4] and it is moreover very direct and simple; there should, therefore, be no difficulty in either confirming it or pointing out precisely the error in it, if there is one. This energy should of course be the same no matter by which process it has been determined; if not, an explanation must be looked for and it ought to be possible to find it, especially if it lies in this simple proof. Fleury's advice in such cases is to "look for the infinity." While this method is new and different from the orthodox ones, it involves no new or unproven laws.

The method is based on the recently discovered and now well known fact that when a current passes through a conductor internal stresses and strains in the form of radial pressures and longitudinal tensile stresses are produced, quite analogous to those in a magnetic field;

3. Trans. A. I. E. E. Vol. 42, 1923, p. 325.

4. A postulate is defined in the dictionary as "a proposition accepted *without proof;* something that must be *assumed* in order to account for something else." (Italics are the author's.)

but as the former act directly to tend to move the material of the conductor, they produce truly ponderomotive forces (tending to move masses), as is readily shown when the conductors are liquid and the current is large enough, as they increase with the square of the current, like most other electromagnetic forces. Being true mechanical forces they can be correctly specified and measured in dynes. Hence by letting these known mechanical forces act through known distances, the corresponding amounts of energy may be determined. Unfortunately neither Ampere nor Maxwell had the advantage of any knowledge of these internal stresses.

Internal Stresses

Many years ago the writer noticed these stresses when large currents were passed through liquid conductors in electric furnaces; the forces were sometimes strong enough to completely rupture the circuit by crushing it radially or tearing it longitudinally, and as this always occurred in one place by depressing the liquid where the cross section of the open channel happened to be least, the phenomenon was colloquially called the "pinch effect" by which term it is now generally known. These forces are now used very effectively in hundreds of electric furnaces for lifting and circulating the liquid metal.

Later, Dr. E. F. Northrup, to whom the writer had described this peculiar phenomenon, developed the mathematical formula for the quantitative value of these pressures, in a very able paper.[5] He based it on the attraction of the filamentary conductors for each other, and also on the radial force acting on each filamentary conductor due to its being in the magnetic field inside of the conductor; both methods gave the same result. It is important to note that it was also confirmed by him later experimentally to a high degree of accuracy (using, of course, a complete circuit), showing that the pressures were true, mechanical ones, and that their units in the formula are in dynes per sq. cm. This formula or law may therefore be safely accepted as quantitatively correct, reliable and accurate. The present writer showed later that in the c. g. s. system this law is a *unit relation* for the pressure at the center of a round conductor which, though it may not be a proof, is always strongly confirmatory of correctness. This easily remembered relation is $p = i^2/S$ in which S is the section.

Northrup's general law for the radial pressure p in dynes per square centimeter at any point in the interior of a round conductor of radius R at a radial distance r from the center, and carrying a current i, is $p = i^2 (R^2 - r^2) \div \pi R^4$, the now well known "pinch" pressure formula; all the quantities are in c. g. s. units. This gives the *mechanical* stresses or strains in the form of radial pressures in any part of the cross section of a round conductor, produced by a current flowing through it. The curve of these pressures is a parabola, Fig. 2; hence for the whole section the curved surface of the loci is a paraboloid of revolution. This mechanical pressure is zero at the circumference, a maximum at the center, and the mean over the whole section can readily be shown to be half of the maximum; the formula also shows that this radial pressure is independent of the length of the conductor. This formula is based on the condition that, as Northrup himself states it, the conductor is a part of a very long straight conductor of circular section "very far separated from its return conductor," hence is not limited to the impossible infinitely long conductor.

Proofs

There are several proofs of this constant, $i^2/2$ ergs per cm. all of which lead to the same result and therefore confirm each other. They are rigid, being free from approximations, dropped factors, infinities, inductance, self-inductances, or mere postulates. One is based on the radial pressures, one on the longitudinal force, and other shorter ones based on what some may consider allowable assumptions. The energy referred to is that which is required to start a steady current; it is constant while the current is flowing, and is set free again when the current stops; it is quite independent of the energy which may be transmitted while the current is flowing; it is quite analogous to the vis viva or the $m\,v^2/2$ energy in a moving body, like that required to bring the cable itself up to its normal speed in a cable transmission. By the first method, based on the radial pressures, it is measured as it is set free, while the current decreases to zero; it involves only two of Maxwell's undisputed laws. By the second method, based on the longitudinal force, it is measured as it is being stored by a constant current in an increased length of the conductor; it involves only the Northrup and the so-called Kelvin laws. In both, the conductor is assumed to be a liquid, and the energy is measured by the mechanical work done by this energy.

Radial Pressure Method. A steady current is assumed to have been started by an external source, in a circuit of zero resistance, hence would continue to flow without a connected source until all its vis viva or stored energy has been consumed by transformation into some other form of energy. This stored energy is quite analogous to the $m\,v^2/2$ energy stored in a body moving at a constant velocity and without encountering any resistance.

An experimental proof that the energy stored in a body moving at a constant velocity is equal to that given by the well known formula $m\,v^2/2$, might be obtained by opposing the motion of that body by a *constant* pressure or force until it comes to rest, when its energy is exhausted; then the product of this force and the distance over which it was applied before the body came to rest, would evidently be equal to this

5. *Physical Review*, June 1907, p. 474.

$m\,v^2/2$ energy, thereby proving that this expression gives the correct amount, or that this amount might be determined in that way if that formula were unknown.

The present method is quite similar. Assume any given length l of such a straight, single, liquid conductor of circular section having a radius R and a current I (c. g. s. units); or it may be the whole circuit of length l, if only it is large enough that each unit length is far enough from the return circuit not to be affected by it. It is well known and could easily be shown, that such a liquid conductor will tend to shrink radially, due to the pressures produced by this stored magnetic energy.

From Maxwell's $H = 2\,I/R$ for such single conductors, and his $H^2/8\pi$ pressure formula, this radial magnetic pressure on the outside surface is easily shown to be $I^2/2\pi\,R^2$ in dynes per sq. cm.; this is the total, *resultant*, pressure; it will be explained below why this is the resultant and under what conditions Maxwell's $H^2/8\,\pi$ pressures are true mechanical pressures in dynes per sq. cm. Whatever may be the detailed explanation of the mechanism of this shrinkage, say through a radial distance d, it is true in any case that this pressure must have then acted through this distance d and has thereby done work. Let the outflow of this liquid due to this shrinkage be assumed to be opposed by a constant mechanical pressure on the liquid (analogously to the constant pressure referred to above opposing a moving body), as for instance by making the liquid which is ejected by this shrinkage raise a weight, as it does in hundreds of electrical furnaces in daily use.[6] This opposing constant outflow pressure is made equal to the above radial pressure.

In thus acting on the outside through the distance d this constant radial pressure has set free a known part of the energy originally stored in the flux; this is determined from the distance d and the known force, equal to this constant pressure multiplied by the mean area. Hence there is then left less flux energy and therefore of course also less current. Let this shrinkage at constant outside pressure against an equal, constant, outflow pressure, continue until the conductor has shrunk to a line, that is, to zero.

At these radial and outflow pressures, assumed to be constant and equal, it can be shown that $I^2/i^2 = R^2/r^2$ in which i and r are the current and radius after a shrinkage; for the first pressure P is $I^2/2\,\pi\,R^2$ and the pressure p after this shrinkage to a radius r is $i^2/2\,\pi\,r^2$; when these are made equal to each other the above relation follows. Hence $I/i = R/r$, that is, the currents will diminish in proportion to the radius, and therefore the current, and with it of course the energy also, will become zero, when the radius has shrunk to zero. This shows that *all* the stored energy has thus been consumed in crushing the conductor to zero. The remaining energies are proportional to the squares of the currents or radii, but this is not an essential relation in this proof.

It also follows that for any given length l of the conductor the total radial *force*, as distinguished from pressure (force = pressure × area), diminishes in proportion to the radius, that is, $F/f = R/r$ and it is therefore also zero when the radius is zero. These simple relations hold only when the outflow pressure is made numerically equal to this constant radial pressure; this outflow pressure always exists at a distance from the center equal to the outside radius divided by the square root of 2, as shown by the Northrup formula, and his experimental demonstrations show that these forces are true, mechanical forces in dynes agreeing quantitatively with Maxwell's formulas.

As the original radial pressure is $P = I^2/2\,\pi\,R^2$ the force at first is $F = 2\,\pi\,R\,P\,l = l\,I^2/R$ in dynes, and as it diminishes in proportion to the radius, the work done by its acting radially to the center is the mean of the original and zero, hence is $l\,I^2/2\,R$, which acting through the distance R gives as the original stored energy $W = l\,I^2/2$ ergs or $I^2/2$ ergs per cm., *which is the basic, fundamental constant sought for.* This quantity was thought by our forefathers to be infinity, as deduced from the "complete circuit" system; the explanation why there is this disagreement will be given below. Attention is called to the fact that this total $i^2/2$ ergs, is just twice that long known to reside in the inside of the conductor.

The same result could be obtained by the calculus, and the liquid might then be assumed to be ejected continuously at the center where the pressure will vary from twice the radial pressure at the start, to zero at the end; the opposing outflow pressure must then be assumed to vary accordingly in order that at every moment there is equilibrium. The radial pressures will then no longer be constant but will decrease, and the current will decrease faster than in proportion to the radius. It is then analogous to stopping the movement of a body by means of a variable, instead of a constant, force.

Longitudinal Force Method. A steady current is assumed to be kept flowing in a liquid conductor of zero resistance, zero weight, and constant diameter, by a continuously applied source. Instead of allowing the mechanical stress to act radially to set free the stored

6. If necessary to picture the details (though they are immaterial and do not affect the theory involved) let the liquid be supposed to be ejected through a tube leading to a cylinder foreign to the conductor, having a piston which raises a constant weight; as the pressures in the interior of the conductor are known to be different at different distances from the center, this tube is assumed to be applied at that radial distance r from the center (namely when $r^2 = R^2/2$) at which this particular pressure exists and is constant during the shrinkage, hence it must be assumed to be moved toward the center as the conductor shrinks. When this is done, and only under those conditions, the quantitative mathematical relations become extremely simple, as will be shown. This shrinkage against an opposing pressure must of course be assumed to take place simultaneously throughout the whole of that circuit, though only a portion of it needs to be considered mathematically.

energy as before, it is now allowed to act longitudinally to lengthen the conductor of constant diameter by a specified amount, whereby the source adds to the circuit the energy stored in this added length. This added energy is calculated by letting this known longitudinal force lift a known weight through a known height while lengthening the conductor by the latter distance; the source thereby does a known amount of external work, and according to the so-called Kelvin law it then simultanously adds to the stored energy of the circuit an amount equal to the external work done. Hence the energy stored in this added length is equal to this known external work done.

The force is calculated from the pressures given by the Northrup law, which is generally recognized as correct, both relatively and quantitatively, and the pressures have been shown experimentally to be true mechanical pressures in dynes per sq. cm.; it may now be safely classed under the "classic" laws. The

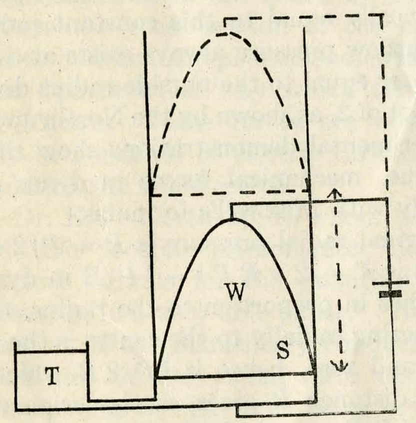

Fig. 3

Kelvin law, though not always referred to by this name, seems to be a universal law in dynamics and when properly worded, may also be safely classed under the "classic" laws. The conditions of the circuit are that only the additional length need be liquid, the parts near it must have the same diameter, the part that moves must be considered as weightless and of course it must be straight for a reasonable length and the return circuit must be far removed.

In Fig. 3, let S be conceived to represent an infinitely thin cross sectional layer of a circular, liquid, conductor of radius R. Let W be a weight, which may be shaped as a paraboloid of revolution to correspond with the pressure loci in Fig. 2, though merely to simplify the conception, as a cylindrical body of half the maximum height and the same base, would have the same weight or volume and would answer quite as well. Let a current i be conceived to be passed axially through this section, the rest of the circuit being shown diagrammatically only. Let the layer S be assumed to be connected at its circumference, where the pressure is zero, to a supply reservoir of more liquid diagrammatically represented by T; the liquid is assumed to be weightless and the supply from the outside is assumed to exert no pressure and to involve no energy.

The conductor S being a liquid the radial pressures of the Northrup formula will act equally well axially, let it be said by hydraulic action, if that is preferred; hence they will raise the weight W, the additional weightless liquid necessary to provide for the lengthening of the conductor, being supposed to be supplied from the outside under no pressure. The pressure formula will be seen to be independent of the axial length, hence these axial pressures will always remain the same as the weight is raised, that is, the total vertical lifting force is a constant, for a constant current. This axial lifting force is being used in hundreds of electrical furnaces in daily use lifting many tons per day, hence is well known and not a mere theoretical force on paper; its quantitative value is also well known.

Let the weight be lifted l cm. (Fig. 3). The surface of the loci of the pressures being a paraboloid of revolution, it can readily be shown[7] that the average pressure over the whole section is half the maximum at the axis. At the axis $r = 0$ (in the Northrup formula), hence the maximum pressure there is $p = i^2/\pi R^2$; multiplying half of this by the total area πR^2, gives $i^2/2$ as the force in dynes; that is, the total lifting force, and therefore the weight W which it will lift, is numerically equal to $i^2/2$ dynes. Having lifted it l cm. the external energy expended on the weight is: energy = force $\times$ distance $= l\, i^2/2$ or $i^2/2$ ergs per unit length. Hence according to the above mentioned Kelvin law, the stored magnetic energy of the conductor per centimeter of length, must be $i^2/2$ ergs also, the source having supplied double this energy. *This is the basic fundamental constant sought for.*

Other proofs. A given radial pressure can always be replaced by its corresponding circumferential tension. If P is the radial pressure on the outside of a conductor as given above, it can be shown that the tensional force (not pressure) in a band encircling a unit length of the conductor and producing the same radial pressure, is $P\, R$ in dynes, in which R is the radius. If a stretched, elastic band, having this tension be assumed to be developed into a straight line (equal to the circumference) and then allowed to shrink to zero length, following the law of the perfect spring, namely that the tension is proportional to its length, it can be shown that the energy set free thereby is again the same, $i^2/2$ ergs.

From the Northrup formula the pressure in dynes per sq. cm. at the center is just twice the $i^2/2\,\pi R^2$ pressure in the same units, at the periphery, from outside. As the outside pressure must also act at the center, it might be argued that the inside flux has added an equal amount, hence that the energies residing inside and outside are equal, therefore the total is double the well known inside energy $i^2/4$.

7. It is well known that the volume or weight of such a paraboloid of revolution is equal to that of a cylinder having the same base and half the height.

At the periphery the two mechanical forces balance, as there is no tendency to movement there, according to the Northrup formula; there is no resultant there. As the forces are equal the energies should be also, as both forces must be considered relatively to the same center, hence as acting through the same distance, the radius. Again the total is twice that inside.

The very basis of the c. g. s. system is that the fundamental relations are unit relations, at least as far as possible. If it may therefore be assumed that the self-inductance (in its true, energy sense) of a unit length of the fundamental conductor, is unity, then if L in the usual expression $L\, i^2/2$ for this energy, is made unity, the energy becomes $i^2/2$. A self-inductance is stated to be physically a length, and in the c. g. s. system it is correctly measured in centimeters. Under fundamental conditions this length and the length of the conductor should be the same thing. This constant then follows directly.

Those who recognize the existence of the longitudinal force, will find that in a fundamental conductor of uniform diameter it is numerically equal to $i^2/2$ dynes, from the Northrup formula, and that in this fundamental conductor it is independent of the diameter or the length. Hence when this force acts to stretch or lengthen its conductor without doing any external work, (though generating a counter e. m. f.) the energy is being stored, just as it would be when the speed of a moving body is increased. If l is this added length, the added energy will of course be $l\, i^2/2$ ergs, which for a unit length again gives $i^2/2$.

Doubtless still more proofs could be found which are also free from the pitfalls of infinities, the ambiguous self-inductances, and the short comings of the complete circuit theories, which had misled us in the past.

Deductions from this Constant

It will be seen that this constant is independent of the diameter of the wire, and depends only on the current, both of which have long been known to be true of the flux energy residing in the interior of the conductor, $i^2/4$. For any length l the energy is $l\, i^2/2$, that is, it is directly proportional to the length. This $i^2/2$ ergs is apparently one of the most basic fundamental constants in electrodynamics, and from it interesting deductions follow.

This $i^2/2$ is the total stored energy, outside of the wire and inside. It has long been known and is easily proved, that the amount stored inside of the conductor is $i^2/4$ ergs per cm. It follows therefore that of the total, half of the energy resides in the inside and half on the outside, as one might expect nature to distribute it.

This constant also shows that the energy of the flux is finite, though the flux itself is of infinite extent, in the same sense that our atmosphere extends to infinity. It is one of those cases in which a *property* of an infinite quantity is finite and therefore affords a means of treating an infinite quantity mathematically without danger of falling into the pitfalls of mathematical infinity. Another finite property of this theoretically infinite flux is what might, in a certain sense, be called the resultant or equivalent flux, as will be described below.

Reduced to the units used in practise the general formula becomes $W = 0.00001524\, l\, I^2$ in which W is the stored energy in watt-seconds or joules, l is the length in 1000-ft. units, and I is the current in amperes. This shows how extremely small it is.

A very interesting and important deduction, showing another new unit relation, is that in the c. g. s. system each unit of current generates one unit of flux around such a conductor, in each unit of length, and independent of the diameter; it includes all the flux inside and outside. But this flux must be specifically defined, as it is a resultant, equivalent, or condensation of all the very large number of lines into which it divides itself as it spreads out into space; these resultant lines might be termed fundamental maxwells. This condensation is such that these resultant lines when combined with the magnetomotive force in common to them all, represent the true stored energy in that field. It is somewhat analogous to supposing our widely diffused atmosphere to be condensed into a thin solid or liquid layer around the earth, which has the same mass. Or these condensed lines may be imagined to be those originally generated by the current and then spread out into space according to the laws of distribution, *but without any change of energy contents,* just as the condensed atmosphere would spread out without change of mass. Such lines are a means of summing up an infinite quantity by one of its properties (energy in this case) to get something finite. They are useful in calculations of flux energy and they clear up some ambiguities, but they are sometimes distinctly different from those entering into calculations of the induction of e. m. fs. by cutting or linking. This will be further discussed below.

The proof of this equality is as follows. The energy residing in a complete circuit of flux is: ergs = maxwells $\times$ gilberts/8 π. The magnetomotive force in gilberts around a single conductor is $4\,\pi\, i$, and as the energy per unit of length is $i^2/2$, it follows that $i^2/2 = 4\,\pi\, i\, f/8\,\pi$ in which f is the number of lines in maxwells; hence $i = f$. This unit relation is a fundamental one and applies rigidly only to the fundamental conductor, and of course, not at all to bi-filar non-inductive circuits, nor when the permeability is not unity. The "complete circuit" mathematics leads us away from it rather than toward it. It seems to mean that in the c. g. s. system current and flux are merely different physical representations of the same quantity, as far as energy is concerned, though, of course, only when the resistance energy is zero; or that magnetism is merely an effect of current at a distance, the energy residing in the moving electrons.

For a unit length and unit radius the total radial force (not pressure) from the outside, is numerically equal to i^2 dynes; hence is another unit relation; for other radii it is inversely proportional to the radius. This force multiplied by half the radius (because it is radial) again gives the constant $i^2/2$.

The stored magnetic energy in any part of a circuit is generally calculated from the formula $L\,i^2/2$ in which L is the self-inductance of that part in centimeters. In the fundamental conductor this energy is $l\,i^2/2$ in which l is the length in centimeters. Hence in such conductors the self-inductance (in the energy sense of that term) is the same thing as the length of the circuit, that is, the distance over which the current flows; another interesting unit relation. This explains why a self-inductance is physically the same kind of a quantity as a length, a purely geometric quantity, at least under the most fundamental conditions; the permeability is of course taken to be unity in all fundamental cases. It also shows why in the c. g. s. system it is correctly expressed in centimeters, or in 10,000. kilometers for the henry. It also follows that under these fundamental conditions the self-inductance is independent of the diameter of the wire, which would seem to follow also from the long-known fact that the energy of the flux in the interior of the wire is independent of the diameter.

This stored electromagnetic energy, $l\,i^2/2$ of a current is quite analogous to the vis-viva or stored mass energy $m\,v^2/2$ of a moving body. In the c. g. s. system both are equal to ½ erg when all the quantities are unity, and one may write $l\,i^2 = m\,v^2$, which means that if all the electromagnetic energy stored by a current i flowing for a distance l, in such a single conductor, be converted into moving mass energy, the relation of the mass to the velocity must be such that $m\,v^2 = l\,i^2$. Thus for say 1000 amperes flowing in such a conductor, the stored electromagnetic energy in every foot, is the same as the mechanical energy stored in a weight of 0.723 or nearly ¾ lb. moving at 1 ft. per sec., or 0.181 lb. at 2 ft. per sec.; all are equal to 0.01124 foot-pounds or 152,400 ergs. This is the energy, per foot of conductor, set free when the current is stopped. For any other length of conductor this equivalent mass increases as this length.

It is of interest to note that for 1 ampere this energy stored per centimeter is equal to that of 5.10 millionths of a gram raised one centimeter, which is extremely small, and shows why the least resistance in such a conductor stops a current almost instantly after the e. m. f. ceases. Yet our forefathers claimed that this energy was infinitely large, which it seems is still being taught.

Another result which this constant, $i^2/2$, has led to, is a better understanding of the true nature of flux energy and calculations pertaining to it, as explained below.

Disagreement

When this stored magnetic energy per unit length of a single conductor is deduced by means of the mathematics of the complete circuit and by some of the older methods and postulates, or by means of self-inductance formulas (always only approximate), the result is that even for a very small current this stored energy per unit length is infinite, while $i^2/2$ is generally quite small. But as the above proofs are simple, brief, rigid, and involve nothing but well established laws, it does not seem possible to find any error in them in a long discussion since the writer's first publication of this result,[8] deduced by a similar though more involved process.

The discrepancy therefore must be looked for elsewhere. The mere fact that the result differs from our older views, cannot of course, be accepted as a proof of an error in it; if that were done in this and other cases, further progress in science would be checked.

The writer believes that this discrepancy can best be located and explained, and that the criticisms of the above proofs can best be answered, by first getting a clearer conception of the various factors and elements involved, and by endeavoring to show how and where some of our older conceptions had misled us, and which of them may be questioned and should be modified or revised if wrong. The new result, if correct, may well be used as a test of the correctness of some of the older laws and postulates.

The Square Circuit

As explained above, a circuit in the form of a very large square (Fig. 4) is the best form of complete circuit for studying the properties of a unit length of a

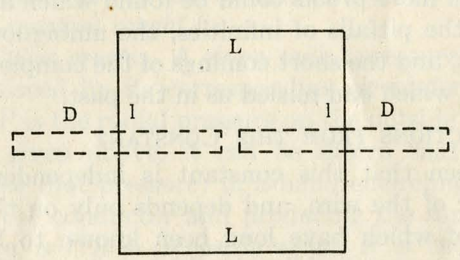

Fig. 4

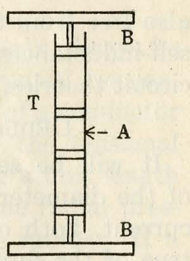

Fig. 5

single conductor. A number of the criticisms involving the "rest of the circuit" are also answered by this form.

If the lengthening of the circuit described in the second proof above, be assumed to take place simultaneously in the middle of two opposite sides, as in Fig. 4, and the material of the circuit be considered weightless, many of the criticisms made will be met. The only additional flux and flux energy will then be in the unit disks D and D; those in and around the rest of the circuit will not have been changed in the least.

8. A Single Straight Conductor as a New Fundamental. *Jour. Frank. Inst.*, Feb. 1925, p. 235. In this article the first part of the paragraph forming the upper half of p. 243 contains an unfortunate arithmetical error and should be deleted.

It is of interest, and answers some criticisms, to suppose the reverse movement to be forced to take place, that is, that even when only a small steady current is flowing in such a circuit as Fig. 4, it be forced to be shortened by a unit length (half a unit on each of two sides) or even less. According to the older views that the stored energy inherent in a unit length of such a circuit (or even less) is infinite, it would have to follow that an infinite amount of energy had been set free by this small motion and small current, which of course would be absurd.

It has been claimed that the other sides $L\,L$ repel each other, thereby tending to lengthen or stretch the others, and that this force *must be added* to that exerted in the unit length. The very premises are, however, that they are so far apart that their repulsion effect on each other along their whole length (which is due to their own flux) may be safely assumed to be zero. It is true, however, that no matter how long the other sides l, l are, the force which lengthened the unit length, caused by the mutual repulsion of the parallel disks of flux, exists along the whole length and therefore acts on L and L also, to repel them at their ends, as they are the abutments. But as this is the *same* force, it would be an evident error to add it to itself.

This may be illustrated by the following mechanical analogy. Let T, Fig. 5, be a tube of any length, even very long; it is conceived to be filled with some elastic material under pressure which is just resisted by pistons acting against the abutments $B\,B$ which therefore seem to tend to repel each other. If now some more of the compressed material be introduced at A the abutments B and B will move apart, but the force that moved them is that applied at A, and must, of course, not be added to the prior force which seemed to repel the abutments, as it is one and the same force. When a rope supporting a weight is lengthened the force in it must not be added to that in the rest, as it is the same force.

Flux and Flux Energy

A unit line of flux, a maxwell, by itself represents no specific amount of energy, any more than an ampere does (after it is started); it might therefore be "energyless" like a wattless ampere, though for different reasons. Amperes represent energy when combined with the electromotive force in phase with them; a maxwell represents stored magnetic energy when combined with its magnetomotive force in gilberts (or with ampere-turns). The energy in ergs is equal to the maxwells multiplied by the gilberts and divided by $8\,\pi$. The magnetomotive force in gilberts in a complete magnetic circuit around a long straight wire, is numerically equal to $4\,\pi\,i$ (i is the c. g. s. unit of current equal to 10 amperes); hence one gilbert $= 10/4\,\pi$ ampere-turns.

But there is a very important difference in the analogy between the amperes in an electric circuit and the maxwells in a magnetic circuit. In an electric circuit the filamentary amperes, each multiplied by their voltage in common and in phase, may correctly be added together to get the total energy; but the writer maintains that this cannot always be done correctly with the maxwells.

The reason for this is best shown by analyzing the flux around and in a straight conductor. In Fig. 6 let C be the cross-section of a conductor; then according to the well known law $H = 2\,i/r$ the ordinates of the Curve F give the flux densities H at any distance r from the center, outside of the wire, while the ordinates of the straight line f give the H inside of it. At the surface the two are, of course, exactly equal, namely h, as they are in equilibrium there.

The flux lines outside may be said to act by their contraction somewhat like layers of stretched rubber bands, the outer ones exerting a radial pressure on

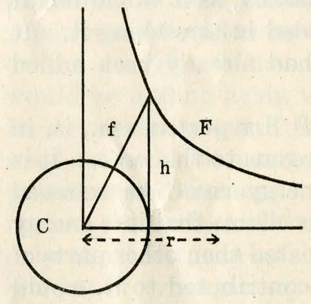

Fig. 6

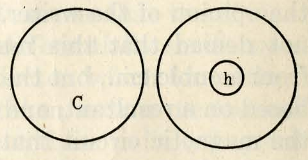

Fig. 7

all those between them and the surface, just as the outer layers of our atmosphere exert a radial pressure on all those below them. Hence the magnetic pressures ($H^2/8\,\pi$ according to Maxwell) at the surface of the wire, are the *resultants* of all those outside of the wire, just as the atmospheric pressure of 760 mm. is the resultant of all the pressures above it. This radial pressure from outside contributes to the flux density H (Curve F) which is greatest at the surface. As H means maxwells per sq. cm. there are true maxwells in all this space around the wire, theoretically all the way to infinity.

But to combine all these maxwells with the gilberts and add all these energies together to get the total, as has been done by some (which of course gives infinite energy) is, in the opinion of the writer neither justified nor correct. It is adding things again that have already been added. The magnetic pressure at the surface is already the sum (integration) of all those beyond, just as the pressure at the bottom of a wall of bricks is the sum or integration of all those at the individual layers above the bottom; it would surely not be correct to add together the total pressures at each layer of bricks to get the total at the bottom.

The stored energy is a function of this pressure; Maxwell's formulas give the pressure in dynes per square centimeter as $H^2/8\,\pi$ and the energy in ergs per cubic cm. as the same $H^2/8\,\pi$. Hence if it is wrong to add the pressures, it is equally wrong to add the energies deduced from them. If one calculates the energy from a *resultant*, one cannot again calculate it for all of the components which make up this resultant and then add them together (or integrate) to get the total. It is either one or the other but not both.

If for instance a rubber rod of a square centimeter section, were stretched say a centimeter, the stretching force would exist in all parts of its length, and energy would be stored in every centimeter of its length. But if it be then cut, and contracts for one cm., all its stored energy would be released in the cubic centimeter space in that cut, even though it had formerly been distributed along its whole length; it is the energy of the *resultant*; the cut could be made anywhere along its length with the same result. To then multiply the energy in that cubic centimeter by the length of the stretched rod to get the total stored energy while stretched, would evidently be wrong, as it would be far greater than the energy expended in stretching it. It would be adding again what had already been added in the resultant.

Maxwell's energy formula $H^2/8\pi$ per cu. cm., is, in the opinion of the writer, analogous to this case. It is not denied that this much energy could be released from a cubic cm., but the writer claims that it is energy based on a resultant, and if released then other parts of the magnetic circuit that had contributed to it, would lose their energy also. What has misled us is that constant current has generally been assumed, in which case any energy taken out of a system is at once restored again by the source. In studying such fundamental cases one should conceive, as stated above, the conditions of zero resistance at which a current once started continues without a source of supply of new energy. If in such a case a part of the stored magnetic energy is set free, the current will fall as was described above. The squares of the currents will be proportional to the remaining energies.

The following analogy (though not quite perfect) will help to understand the physical nature of these magnetic stresses, and will perhaps help to clarify some disputed points. Let C Fig. 7 be the section of a long cylinder with a very thick wall and an infinitely small hole along its axis; let it be conceived to be made of something very elastic like rubber, in the sense that rubber is elastic. Let water be forced into the hole until it expands into the large hole h; the ends are closed, and may be neglected in considering a unit length in the middle.

The compressive and tensile strains or stresses in the wall around h will then be closely analogous to the magnetic strains or stresses in the ether around the outside of a straight conductor, h representing the conductor. They will be greatest at the surface of the column h, decreasing to practically zero on the outside surface of the rubber. If the mechanically elastic properties of the rubber were the same as what might be called the magnetically elastic properties of the ether, (Maxwell's statement to the effect that the tensile stress along lines of force is equal to the repulsion between like lines, seems to define these elastic properties), the analogy would be more perfect, as far as the stresses and energy are concerned; but it does not extend to the water, as the stresses in it are quite different from those in the inside of the conductor; nor does the conductor have to collapse to release the magnetic energy, it is the current in it which becomes zero when the field becomes zero (for zero resistance). A complete mechanical analogy does not seem to exist.

The energy stored in this stressed rubber per unit length is certainly *finite*, this cannot be questioned, and it is exactly equal to that which was required to force the water into the hole; it therefore cannot possibly be infinite. The radial pressure at the surface of the hole h is the *resultant* or summation of all the components beyond it; when it is allowed to act, as by letting it force out the water, the stresses in other parts disappear also, as they formed a part of the total. If the energy is set free by the action of the resultant of component forces, it evidently cannot then be set free again by the component forces also; either one or the other but not both, can act to set free the energy. To integrate these pressures in successive infinitely small, concentric layers would be summing up what had already been summed up before, hence would give erroneous results, far too great. Hence integrating the energies in such layers, based on the pressures existing there, would be equally absurd, yet this is what has been done in the electrical case as an alleged proof to show that the stored energy per unit length is infinite.

The above seems to show that, while there are lines of flux, maxwells, in the space surrounding a wire, it is not proper (as has been done) to combine all these maxwells with their magnetomotive forces, to get the energies represented by them, and then add all these energies to get the total. Hence it seems to be proper to apply to some of them the corresponding term used with amperes, and say they are *wattless* (to some extent) in the sense that they cannot all represent the energy attributed to them when considered individually. To do so would be to add to the energy of a resultant the energy represented by the components which make up this resultant. When a resultant exists, the energy in a system is that represented by this resultant, and no more.

Amperes are wattless when out of phase with their e. m. f. while maxwells are wattless when their energies are represented by or attributed to, that of their resultant. The e. m. f. induced by the linkage or cutting of lines, is independent of whether they are wattless or not, as will be explained below.

There being no such thing as a magnetic insulator, the lateral spreading of flux due to the repulsion of like lines, can be resisted only by other flux; the magnetic properties of iron are, of course, always excluded in a study of fundamental laws and properties. The radial magnetic pressure from the outside, at the surface of a round wire, must therefore be exactly balanced by an equal and opposite radial magnetic pressure from the inside, due to the lateral pressure of the inside flux. This is known to be the case and follows from what was shown in Fig. 6, in which the densities H just outside and inside of the surface, represented by the ordinate h, are equal, and therefore the pressures $H^2/8\pi$ in dynes per sq. cm. in opposite directions, are also equal and therefore in equilibrium. Hence during the steady state there is no resultant radial pressure between these two at this surface, though this surface, considered as a thin layer, may be said to be compressed magnetically as though between the jaws of a vise. This is confirmed by the Northrup formula in which the mechanical radial pressure at the surface, when $r = R$, is zero. This has been the subject of some discussion, as it was thought that the tensile stress of the outside lines ought to exert a radial mechanical pressure from the outside. When a weight is supported on a hand there is no resultant force which can do any work, the upward and downward forces being equal. But when the upward pressure is reduced by only the slightest amount, the weight will descend and do work equal to the weight multiplied by the distance. Similarly when the pressure in the inside of a conductor is slightly relieved, as for instance if it were a liquid, part of which was forced out, then the outside pressure will act. The conditions are somewhat analogous to those in a lever.

The inward pressures from the outside flux are balanced by the outward pressures of the inside flux; hence the former are represented by the latter, and are transferred to that flux, and it is in this interior flux that the conductors tend to move to the center, thereby exerting the pressures in the Northrup formula. It is for this reason that the mechanical pressures on the inside, and therefore the energy deduced from them, are based on the total pressures, inside and outside, and therefore represent the total energy.

Maxwell's $H^2/8\pi$ Pressures. Doubt has often been raised as to whether these $H^2/8\pi$ pressures and tensions are really in dynes per sq. cm. for if they are, they are real, ponderomotive or mechanical pressures, and not mere mysterious magnetic pressures. The fact that these magnetic pressures exist radially on the outside of a conductor and yet there does not seem to be any mechanical force there, under stable conditions, has been explained above. Northrup's experimental measurements of the actual mechanical forces in the interior of a liquid conductor, produced by these theoretical electromagnetic forces, demonstrated that the latter are correctly expressed in dynes per sq. cm. Hence these $i^2/2\pi R^2$ radial pressures from the outside are true mechanical pressures provided they are properly interpreted.

To be mechanical these $H^2/8\pi$ stresses must of course act on some mechanical abutment; the ether cannot act as one. The lack of proper consideration of such abutments no doubt has given rise to much confusion; they do not appear as factors in calculus calculations. The two unlike poles of nearby magnets are the abutments for the tension of the lines between them. The only abutment of the endless lines in space encircling a conductor is, of course, the surface of the material conductor against which they exert radial pressures, as a stretched rubber band would do.

In the writer's opinion this $H^2/8\pi$ expression can be applied correctly to parts of fields only when the density is uniform, which it can be only when the lines are straight and parallel; the pressures of none of them are then resultants of the pressures of others. Even then they cannot be added, as by integration, any more than one could add together the pressures at successive layers of a confined compressed gas. When the lines are curved, those on the inside of the curve support the pressures of those outside of it besides their own, as superimposed rubber bands would; hence to add them, or the energies deduced from them, by integration, would be adding again what had already been added. In differentially small parts of fields uniform density may no doubt be assumed. It seems that the $H^2/8\pi$ pressures and tensions in the differentially small layer next to the surface of the conductor, which forms their only abutment, can safely be taken as true mechanical stresses in dynes per sq. cm. but those radially beyond must not be added to them.

Induction

Attention is called to the fact that the e. m. f. induced by cutting or linking a definite number of lines of flux per second is entirely independent of the energy inherent in that flux; hence conversely it is not proper to draw any conclusions as to the energy in that flux from this induced e. m. f.; yet this has been done.

Imagine a field between two parallel rectangular iron pole faces; let there be F lines in this field and let it be cut by or linked with, a moving circuit, in a given time. Now let the pole faces be doubled in length or width but contain the same number of lines F and be cut or linked in the same time as before. The induced e. m. f. will be exactly the same, yet these same lines in the first case will have four times the energy in them that they had in the second case, as their density H is twice as great. Yet in the case of self-inductance, from the counter e. m. f. induced by a rising current, very positive conclusions have been drawn concerning the energy inherent in the self-produced flux lines that have been cut or linked.

This, in the opinion of the writer, is not justified for the reasons just given, even though it is generally accepted and important conclusions are based on it. It seems that this is partly the explanation of the disagreement between the writer's results and those based on mere "postulates" and on some of our older theories.

It is true that in the case, for instance, of a large single-turn circuit, without iron, of course, the ampere-turns are the magnetomotive force of the flux circuits around the wire and linked with the loop; also that the energy of flux is proportional to the product of the maxwells and the ampere-turns; but in the opinion of the writer the latter is not true when the energy in some of the flux lines is the resultant of the energy of some of the others, that is, when the pressures of some lines are components of the pressures of others, hence some of them must be considered as wattless in the sense above described; if not, as explained above, it would be adding again what had already been added before.

In a very large single-turn circuit for instance, the flux encircling a unit length of circuit would not differ greatly from that around a unit length of the fundamental straight wire, in that the flux distant from the wire exerts a pressure on that nearer to it, and that the pressures of those lines nearer the wire are therefore resultants of those beyond. The e. m. f. induced by the linkages or the cutting of such wattless lines, is the same as with the others, but the energies represented by them are not. Hence it does not appear to be justifiable to draw such definite conclusions as have been drawn about the energy of the flux lines, from merely the e. m. f. induced by them, and the m. m. f.

Self-Inductance

In the writer's opinion this term self-inductance has been used in two different senses, one when applied to energy and the other when applied to the induction of a counter e. m. f. in its own circuit when its own current varies.

When the energy per unit length of a circular circuit of say one turn, is calculated (as it generally and perhaps always has been) from the self-inductance as determined from the usual formulas, it becomes larger as the radius increases and is infinitely large for a straight conductor considered as a circle of infinite radius. But all self-inductance formulas for such circles are only approximate (some factors having been dropped as too small), and seem to involve empirical constants. To extrapolate an approximate formula to infinity is evidently not rational. Moreover in extrapolating even a rigidly exact expression to infinity the result may sometimes have been interpreted as infinity when it really is an indeterminate, a suppressed or dropped factor may make this difference.

Hence the explanation of the disagreement of the writer's result with that obtained in the older way from such formulas, is better looked for in the latter which involves infinities, as the method used in the former is mathematically rigid, and involves no infinities or indeterminates. These self-inductance formulas have no doubt been tested experimentally and have been found to be (or have been adjusted to be) correct to limited extents, but this applies only to the range of circuits tested, beyond which it is still an extrapolation, hence involves the pitfalls of that process. Within the ranges tested, the self-inductance per unit length no doubt increases when the diameter of the circuit increases, but the writer maintains that when the circles become very large it must start to decrease again reaching the above theoretical limit for a straight conductor. It is of interest that for the limit in the other direction, the smallest possible circuit (the torus referred to in the beginning of this paper), the usual formulas approach quite closely to the above theoretical limit.

If a flexible wire is curved, a current in it will tend to straighten it; this is due to the greater pressure of the disks of flux (due to a greater density H) against each other on the inner side of the curve; or to the longitudinal force which tends to stretch and therefore straighten it; Maxwell's pressure formula $H^2/8\pi$ shows this greater pressure, the abutments of the force being of course in the wire as the ether cannot act as an abutment. By this straightening it expends a part of its stored energy; hence it must have less when straight than when curved as claimed above, even though according to the approximate self-inductance formulas (though beyond their tested ranges) it ought to have more then. As this would take place with zero resistance, when a current once started will continue to flow without connection to any source, this straightening is done at the expense of its stored energy and it is therefore not a case under the Kelvin law, there being no source to supply additional energy.

Even for very large circuits the energy calculated from these approximate formulas is not enormously greater than the writer's small result and it is *nowhere near infinity*; hence it would take but a small change of curvature of the curve to make them correspond for a straight wire or a very large circle. In a small circuit the "return" part of the circuit tends by induction to *increase* the current in the other part; by the very definition of the fundamental conductor, this kind of induction no longer exists. As a circuit becomes larger this induction may become larger due to increasing length, but it diminishes as the square of distance, hence must *decrease* as the circuit becomes larger.

The error pointed out above, due to adding the pressures of components to those of their resultant which already contains them, seems to grow less as coils become smaller and especially when they are solenoids; it may, therefore, be small in the coils used in practise, which may explain why it was not noticed before. It may nearly disappear in the torus (a solenoid with a circular axis) in which the field is nearly uniform. When quite uniform this error naturally disappears.

Self-inductance is usually defined as flux per ampere, hence as a *derived* quantity and not as an independent one, which it seems to be in the energy sense. This definition makes it dependent on only the flux and the current, and nothing else. It is known, however, that in practise it is dependent also on such factors as frequency, shape, skin effect, phase difference, etc. From the physical dimensions it is true that flux

divided by current is a length; and in the fundamental conductor the "resultant" flux (in the energy sense) is the product of the current and the length of its path in c. g. s. units.

A resistance (which is physically an independent quantity) was formerly considered to be defined correctly as a derived quantity, namely volts per ampere, but it is now known that although this is sometimes correct (for d. c.), it may be so very wrong (when the volts and the amperes are not in phase) that this derived quantity was given a special name, impedance. The wattless ampere resulted from this distinction. Today we can realize the confusion and misfits that must have existed before this distinction was recognized by Kennelly over 30 years ago. But the true quantity, called self-inductance, is a purely geometric one, a length, hence is really an independent quantity, like a resistance, independent of any flux or current, hence why should it be defined as *derived* from these; the only reason seems to be, that like in the case of resistance this quotient *sometimes* gives the correct result. But again, as in the case of resistance as a derived quantity may this not sometimes give incorrect results; may there not be a difference between two self-inductances analogous to that between resistance and impedance; and if so, would this not lead to wattless flux (as described above) just as the difference between resistances and impedance has led to the wattless ampere?

The counter e. m. f. in self-inductance induced by the self-generated flux of a varying current depends only on the number of lines cut or linked and is independent of whether they are wattless or not, as was explained above. The lines in the interior however can have but a fractional value of those outside, as they do not cut the entire conductor.

The writer believes that the disagreement between his result and that deduced by the older views, is due largely to the fact that self-inductance had been defined as a derived quantity and that a similar distinction should be made as was done in the case of resistance. That kind of self-inductance which it is always correct to use in energy calculations, should be distinguished from the kind which it is always correct to use in inductance and for the latter the term self-inductance might well be retained as it describes it correctly. Sometimes these two self-inductances may be equal, as in the case of resistance and impedance, but sometimes they may differ, and then some wattless factor is involved. There seems to be nothing unreasonable in this.

The Longitudinal Force

In the second proof above it was assumed that the radial forces, (which, being perpendicular to the conductor are in accordance with past teachings and therefore have been accepted), acted hydraulically to produce the axial forces which lifted the weight. For many years[9] the writer has maintained (as Ampere did) that there exists also a force in the direction of the axis, hence a longitudinal force, which, acting in both directions, he termed a "stretching" force tending to elongate the conductor. Maxwell in Art. 526 discusses it briefly and does not there deny its existence. The writer showed by experiments[10] that this force acted also in solids, hence can not be only hydraulic; also that theory demanded it; the newer electronic theories also seem to demand it. This force is due to the mutual repulsion of the lines of force encircling a conductor, which forces have their anchorage or abutments in the material of the conductor, the only place where they could be. The existence of this force, which cannot be denied, is now receiving more general acceptance.

In the opinion of the writer the recognized perpendicular force and this disputed longitudinal one, are in fact really the same force, as the ether seems to act like a liquid or gas, in the sense that a magnetic force exerted on it in any direction, acts in all directions, as a mechanical force does in liquids and gases, in which the parallelogram of forces no longer applies. Maxwell's conclusion that the tension along lines is numerically equal to the repulsion between them, seems to bear this out.

Ampere's Method Revived

That great mathematical physicist, Ampere, endeavored to develop a mathematical treatment of electric circuits based on straight elements of conductors. Maxwell (Art. 526) did not approve it and preferred his complete circuit system, which has been very useful; but it has unfortunately led us away from the Ampere system, the development of which has been neglected. Ampere had not the advantage of knowing anything about the more recently discovered internal stresses in a conductor (the so-called pinch effect), which has led to the above basic constant, and which, together with the Ampere system, leads to a possible mathematical treatment of electric circuits, which while not replacing the Maxwell system, should be developed as an alternative system to use when the Maxwell system is inadequate, or misleads (due sometimes to infinities) or fails to disclose important factors, such as this constant and the wattless elements. Many interesting and useful developments seem to be possible now in such a system of mathematical treatment in which the fundamental is the single conductor (Ampere's method), in place of the complete circuit of Maxwell, and some of the results in the former could not have been deduced from the complete circuit system.

Conclusions

In the opinion of the writer, the above proofs, deductions, and discussion, will show that a new system of treatment of electrical problems can be based on the

9. The Stretching of a Conductor by Its Current. *Jour. Frank. Inst.*, Jan. 1911, p. 73. Also other later papers.

10. Electromagnetic Forces, Trans. A. I. E. E., Vol. 42, 1923, p. 311, see more particularly Figs. 6 and 9.

single, straight, conductor, as distinguished from that based on the Maxwell complete circuit; not to replace the latter system but to supplement it, and to test the correctness of parts of it. The single conductor system leads to some new and useful results and shows that we should modify some of our former conceptions; also that in flux lines there is an analogy to the wattless ampere which ought to be recognized. It also shows that the term self-inductance has been used in a dual sense and that a distinction should be made analogous to that between resistance and reactance. Some heretofore unknown and useful relations have been deduced from what is believed to be one of the most fundamental constants in electrodynamics, the value of which is determined by simple proofs. Some of the results deduced could not have been deduced from the complete circuit system, which has, in some cases, been misleading.
